

DIMENSIONLESS PHYSICAL-MATHEMATICAL MODELING OF TURBULENT NATURAL CONVECTION

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ABSTRACT

Natural convection heat transfer is present in the most diverse applications of Thermal Engineering, such as in electronic equipment, transmission lines, cooling coils, biological systems, etc. The correct physical-mathematical modeling of this phenomenon is crucial in the applied understanding of its fundamentals and the design of thermal systems and related technologies. Dimensionless analyses can be applied in the study of flows to reduce geometric and experimental dependence and facilitate the modeling process and understanding of the main influence physical parameters; besides being used in creating models and prototypes. This work presents a methodology for dimensionless physical-mathematical modeling of natural convection turbulent flows over isothermal plates, located in an “infinite” open environment. A consolidated dimensionless physical-mathematical model was defined for the studied problem situation. The physical influence of the dimensionless numbers of Grashof, Prandtl, and Turbulent Prandtl was demonstrated. The use of the Theory of Dimensional Analysis and Similarity and its application as a tool and numerical device in the process of building and simplifying CFD simulations were discussed.

Keywords: nondimensionalization, natural convection, physical-mathematical modeling, turbulence modeling

NOMENCLATURE

$CD_{\kappa\omega}$	$\kappa - \omega$ SST parameter	\bar{p}	modified relative turbulent transport pressure in the model RANS, Pa
\hat{C}_p	heat capacity at constant pressure, J/(kg.K)	P_{atm}	atmospheric pressure, Pa
$C_{\varepsilon 1}$	empirical constant of $\kappa - \varepsilon$ model	Pr	Prandtl number
$C_{\varepsilon 2}$	empirical constant of $\kappa - \varepsilon$ model	Pr_t	turbulent Prandtl number
C_μ	empirical constant of $\kappa - \varepsilon$ model	\mathbb{P}	dimensionless modified relative turbulent transport pressure in the model RANS
\mathbb{E}	dimensionless dissipation rate of turbulent kinetic energy	T	temperature, K
F_1	first blending function of $\kappa - \omega$ SST model	T_{REF}	average reference temperature, K
F_2	second blending function of $\kappa - \omega$ SST model	\bar{T}	average component of temperature in the RANS model, K
g	gravity acceleration, m/s ²	T_p	plate temperature, K
Gr	Grashof number	T_∞	free-stream temperature, K
k_t	thermal conductivity, W/(m. K)	\bar{u}_i, \bar{u}_j	indicial velocity average components in the RANS model, m/s
\mathbb{K}	dimensionless turbulent kinetic energy	\bar{V}_i, \bar{V}_j	Dimensionless indicial velocity average components in the RANS model
L_p	plate longitudinal length, m	\mathbb{W}	dimensionless specific dissipation rate of turbulent kinetic energy
\bar{p}	mean relative pressure in the RANS model, Pa		

x_i, x_j	indicial coordinates, m
X_i, X_j	dimensionless indicial coordinates
y	the distance from the field point to the nearest wall, m
Y	dimensionless y coordinate
z	elevation, m
Z	dimensionless z coordinate

Greek symbols

α	thermal diffusivity, m^2/s
α^*	coefficient combination function of $\kappa - \omega$ SST model, in α_1^* and α_2^* forms
β	thermal coefficient of volume expansion, K^{-1}
β^*	$\kappa - \omega$ SST empirical constant
δ_{jz}	Kronecker's delta function
β'	$\kappa - \omega$ SST empirical constant
ε	dissipation rate of turbulent kinetic energy, m^2/s^3
θ	dimensionless temperature
κ	turbulent kinetic energy, m^2/s^2
μ	dynamic viscosity, $kg/(m.s)$
μ_t	turbulence dynamic viscosity, $kg/(m.s)$
ν	kinematic viscosity, m^2/s
ν_t	turbulence kinematic viscosity, m^2/s
ρ	fluid density, kg/m^3
$\bar{\rho}$	fluid density evaluated in \bar{T} , kg/m^3
σ_ε	empirical constant of $\kappa - \varepsilon$ model
σ_κ	empirical constant of $\kappa - \varepsilon$ model
σ_κ^*	$\kappa - \omega$ SST empirical constant
σ_ω	$\kappa - \omega$ SST empirical constant, in $\sigma_{\omega 1}$ and $\sigma_{\omega 2}$ forms
ω	specific dissipation rate of turbulent kinetic energy, s^{-1}

INTRODUCTION

Natural convection occurs due to buoyancy forces, which are due to the combined effects of density gradients in the fluid due to gravitational forces (caused by temperature gradients or concentration gradients) and (Bergman *et al.*, 2014). Compared to forced convection, the natural convection differs by the strong mathematical coupling of the temperature field with the flow field and by the lower heat transfer coefficient by convection and consequent higher thermal resistance (Bejan, 2013).

Natural convection cooling systems increasingly play a greater influence on the operational control and safety of temperatures in electronic and power generating devices (Bergman *et al.*, 2014).

More and more studies and researches are being carried out by the industry and academic community to understand the process, to better model natural convection phenomena, obtaining more realistic solutions. These studies aim to optimize thermal efficiency and increase the applicability of cooling systems exclusively by natural convection in current technologies. As examples, one can mention the following works: Kitamura *et al.* (2015) conducted an experimental study of natural convection from

upward-facing flat plates, presenting a survey of empirical correlations of the Nusselt number versus the Rayleigh number, for the laminar and turbulent regimes; Frank *et al.* (2019) analyzed the combined cooling of electronic components by natural convection with thermal radiation; Verdério Júnior *et al.* (2021) presented a numerical-experimental analysis of the parameters that influence the natural convection heat transfer rates on isothermal square plates, within large cavities; Liu *et al.* (2021) numerically studied steam condensation with air, in natural convection conditions, in tubes bundle channel; Wang *et al.* (2021) performed a numerical-experimental investigation of natural convection heat transfer in supercritical water flow in an inclined smooth tube; Freile *et al.* (2021) developed more accurate correlations of natural convection heat transfer to the reactor pressure vessel cavity cooling system; and Silva *et al.* (2021) who studied two different mesh configurations (at different refinement levels) for numerical evaluation of the turbulent natural convection heat transfer on an isothermal rectangular flat plate.

The present work has as objective to present a methodology of dimensionless physical-mathematical modeling of the natural convection in turbulent regime, defined for the particular case of flow over an isothermal plate (with generic geometry) located in a large open environment. This methodology will define a final mathematical model of the evaluated problem situation, which will subsidize future studies on the subject and in the most diverse applications.

PROBLEM DEFINITION

The problem under study consists of natural convection on an isothermal square plate, with temperature T_p , with generic surface geometry (smooth, corrugated, etc.) and located in a large open external environment with constant temperature T_∞ and atmospheric pressure P_{atm} .

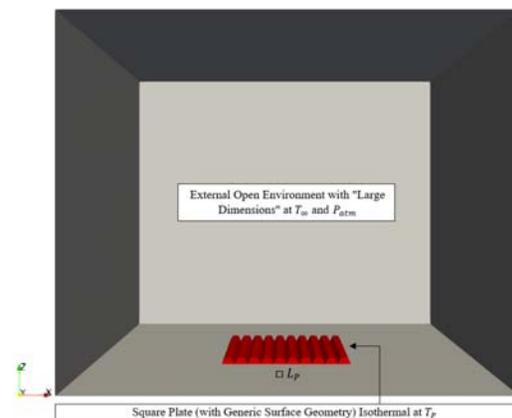


Figure 1. Schematic graphical representation of the studied problem situation.

Here, it is assumed that the temperature difference between the plate and the ambient is high enough to introduce flow perturbation inducing turbulence. Figure 1 illustrates the schematic arrangement of the problem situation studied.

SIMPLIFYING HYPOTHESES

In selecting the governing equations and identifying the physical-mathematical model to be solved numerically, several simplifying hypotheses and physical models were adopted. Based on the physical characteristics of the problem and typical conditions established in Pope (2000), Bird *et al.* (2002), Versteeg and Malalasekera (2007), Incropera *et al.* (2008), Çengel and Ghajar (2012), Bejan (2013), Çengel and Cimbala (2015) and Fox *et al.* (2018), the following conditions were used:

- Steady state flow.
- Air is a Newtonian fluid, and all thermal physical properties (viscosity μ , thermal conductivity k , specific heat C_p) are assumed constant and evaluated at the average reference temperature ($\bar{T}_{REF} = (T_P + T_\infty)/2$).
- Use of Boussinesq's approximation for modeling the buoyancy forces, i.e., density is constant in all terms, except in the buoyancy term, where it can be linearized with temperature based on the thermal expansion coefficient (β):

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_{\bar{P}} \quad (1)$$

, as:

$$\rho(T) \approx \bar{\rho} [1 - \beta(T - T_\infty)] \quad (2)$$

Based on the ideal gas law, $\beta = 1/\bar{T}_{REF}$.

- Thermal radiation heat transfer between the plate and the environment (non-participating or transparent) can be neglected.
- The external environment is admitted "infinite", with dimensions large enough not to exert any physical influence on the plate region's flow and convection heat transfer.
- Flow in turbulent regime, with modeling and treatment using the RANS (Reynolds Averaged Navier-Stokes) Method.
- Turbulence production of κ , ε and ω in their transport equations was considered negligible.
- Use of the SIMPLE algorithm (Semi-Implicit Method for Pressure-Linked Equations) for the pressure-velocity-temperature coupling of the transport equations.

PHYSICAL-MATHEMATICAL MODELING

After identifying the physical-geometric characteristics, simplifying hypotheses, and physical models that were applied to the analyzed problem situation, the physical-mathematical model is defined.

Boussinesq's hypothesis (1897) affirms that turbulent dynamic viscosity (μ_t) is a property of the flow and not of the fluid and that it leads to strong nonlinearities in the transport equations.

From the simplifying hypotheses and physical models adopted and from the Boussinesq's hypothesis (1897), we have, in that order, the turbulent time-average equations of Conservation of Mass, Momentum Balance (called the Generalized Hypothesis of Boussinesq Equation), and Conservation of Energy Principle:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (3)$$

$$\bar{\rho} \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left[(\mu + \mu_t) \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{\partial \bar{P}}{\partial x_i} - \bar{\rho} g_i \beta (\bar{T} - T_\infty) \right] \quad (4)$$

$$\frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j \bar{T}) = \frac{\partial}{\partial x_j} \left[\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{\partial \bar{T}}{\partial x_j} \right] \quad (5)$$

Where the term \bar{P} is called modified relative turbulent transport pressure, defined as:

$$\bar{P} = \bar{P} + \frac{2}{3} \bar{\rho} \kappa + \bar{\rho} |g|z \quad (6)$$

Turbulence Modeling

In solving the mathematical closure problem of turbulence modeling, 2nd order models were used, with two equations and category I: $\kappa - \varepsilon$ and $\kappa - \omega$ SST; both based on the concept of turbulent viscosity (μ_t or ν_t). Such models are formulated and described in greater detail in Pope (2000), Vieser *et al.* (2002) and Menter *et al.* (2003).

The semi-empirical formulation of the $\kappa - \varepsilon$ turbulence model is presented in the following equations:

$$\nu_t = \frac{\mu_t}{\bar{\rho}} = C_\mu \frac{\kappa^2}{\varepsilon} \quad (7)$$

$$\bar{u}_j \frac{\partial \kappa}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\kappa} \right) \frac{\partial \kappa}{\partial x_j} \right] + \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} - \varepsilon \quad (8)$$

$$\bar{u}_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{\varepsilon 1} \frac{\varepsilon}{\kappa} \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} - C_{\varepsilon 2} \frac{\varepsilon^2}{\kappa} \quad (9)$$

Where the empirical constants of this model are equal to $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $C_{\mu} = 0.09$, $\sigma_{\kappa} = 1.0$ and $\sigma_{\varepsilon} = 1.3$.

The $\kappa - \omega$ SST turbulence model combines the robust and precise formulation in the treatment of regions next to solid walls of the $\kappa - \omega$ turbulence model, with the independence of parameters in free-stream and outside the boundary layer of the $\kappa - \varepsilon$ turbulence model. Its semi-empirical mathematical formulation is described below and according to Table 1:

$$\bar{u}_j \frac{\partial \kappa}{\partial x_j} = \min \left[v_t \frac{\partial \bar{u}_i}{\partial x_j} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right); 10\beta^* \kappa \omega \right] - \beta^* \kappa \omega + \frac{\partial}{\partial x_j} \left[(v + \sigma_{\kappa}^* v_t) \frac{\partial \kappa}{\partial x_j} \right] \quad (10)$$

$$\bar{u}_j \frac{\partial \omega}{\partial x_j} = \frac{\alpha^*}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)^2 - \beta' \omega^2 + \frac{\partial}{\partial x_j} \left[(v + \sigma_{\omega} v_t) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \sigma_{\omega 1} \frac{1}{\omega} \frac{\partial \kappa}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (11)$$

$$F_1 = \tanh \left\{ \left[\min \left[\max \left(\frac{\sqrt{\kappa}}{\beta^* \omega y}, \frac{500v}{y^2 \omega} \right), \frac{4\bar{\rho} \sigma_{\omega 2} \kappa}{CD_{\kappa \omega} y^2} \right] \right]^4 \right\} \quad (12)$$

$$CD_{\kappa \omega} = \max \left(2\bar{\rho} \sigma_{\omega 2} \frac{1}{\omega} \frac{\partial \kappa}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-10} \right) \quad (13)$$

$$F_2 = \tanh \left[\left[\max \left(\frac{2\sqrt{\kappa}}{\beta^* \omega y}, \frac{500v}{y^2 \omega} \right) \right]^2 \right] \quad (14)$$

$$v_t = \frac{0.3\kappa}{\max \left(0.3\omega; \frac{\sqrt{2}}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) F_2 \right)} \quad (15)$$

$$\alpha_1^* = \alpha_1^* F_1 + \alpha_2^* (1 - F_1) \quad (16)$$

Table 1. Empirical conditions used in the $\kappa - \omega$ SST turbulence model, according Menter *et al.* (2003).

Constants	σ_{κ}^*	σ_{ω}	β'	α^*
$\alpha_1^* (\kappa - \varepsilon)$	0.85	0.5	0.075	5/9
$\alpha_2^* (\kappa - \omega)$	1.0	0.856	0.0828	0.44

DIMENSIONLESS FORM OF PHYSICAL-MATHEMATICAL MODEL

Definition of dimensionless groups

After the characterization of the physical-mathematical model that describes the problem situation under study, there is the definition and application of the methodology of nondimensionalization of the governing equations.

The nondimensionalization process of the transport equations and turbulence models used begins with defining the dimension and velocity

characteristic of the studied problem. The plate longitudinal length L_p was used as the characteristic dimension of the studied flow. Because of the inexistence of a reference velocity in the study of natural convection, it was decided to use an analogous term of velocity characteristic to the flow, given by the ratio v/L_p .

Thus, from the definitions of the parameters characteristic of the flow studied, there is the definition of the dimensionless groups of interest:

$$X_i = \frac{x_i}{L_p} \quad (17)$$

$$X_j = \frac{x_j}{L_p} \quad (18)$$

$$Z = \frac{z}{L_p} \quad (19)$$

$$\bar{V}_i = \frac{\bar{u}_i L_p}{v} \quad (20)$$

$$\bar{V}_j = \frac{\bar{u}_j L_p}{v} \quad (21)$$

$$\bar{\theta} = \frac{\bar{T} - T_{\infty}}{T_p - T_{\infty}} \quad (22)$$

$$\mathbb{P} = \frac{\bar{P}}{\bar{\rho} \left(\frac{v}{L_p} \right)^2} \quad (23)$$

$$\mathbb{K} = \frac{\kappa}{\left(\frac{v}{L_p} \right)^2} \quad (24)$$

$$\mathbb{E} = \frac{\varepsilon L_p}{\left(\frac{v}{L_p} \right)^3} \quad (25)$$

$$\mathbb{W} = \frac{\omega L_p}{\left(\frac{v}{L_p} \right)} \quad (26)$$

$$\mathbb{Y} = \frac{y}{L_p} \quad (27)$$

Dimensionless Transport Equations

From the application of the dimensionless groups previously defined in the Equations of Mass Conservation, Momentum Balance, and Conservation of Energy Principle – in that order, Equations 3 to 5 – we finally have:

$$\frac{\partial \bar{V}_i}{\partial X_i} = 0 \quad (28)$$

$$\frac{\partial [\bar{V}_j \bar{V}_i]}{\partial X_j} = \frac{\partial}{\partial X_j} \left[\left(1 + \frac{\mu_t}{\mu} \right) \left(\frac{\partial \bar{V}_i}{\partial X_j} + \frac{\partial \bar{V}_j}{\partial X_i} \right) \right] - \frac{\partial \mathbb{P}}{\partial X_i} + \text{Gr} \bar{\theta} \delta_{iz} \quad (29)$$

$$\frac{\partial[\bar{V}_j\bar{\theta}]}{\partial X_j} = \frac{\partial}{\partial X_j} \left[\left(\frac{1}{Pr} + \frac{\mu_t}{\mu} \frac{1}{Pr_t} \right) \frac{\partial \bar{\theta}}{\partial X_j} \right] \quad (30)$$

Where the term dimensionless \mathbb{P} is calculated using the expression:

$$\mathbb{P} = \frac{\bar{P}}{\bar{\rho} \left(\frac{v}{L_p} \right)^2} + \frac{2}{3} \mathbb{K} + \frac{Gr \cdot Z}{\beta(T_p - T_\infty)} \quad (31)$$

From the resulting dimensionless governing equations, the physical influences of the Grashof number (Gr), Prandtl number (Pr) and turbulent Prandtl number (Pr_t) on the modeled flow are evident.

The Grashof number measures the ratio between buoyant forces and viscous forces, according to Incropera *et al.* (2008). It can be used in the classification of flows in natural, forced or mixed convection and is calculated using:

$$Gr = \frac{|g|\beta(T_p - T_\infty)L_p^3}{v^2} \quad (32)$$

According to Incropera *et al.* (2008), the Prandtl number measures the ratio between momentum and thermal diffusivities. It represents how effective is the diffusive transport of momentum and energy through the flow. Calculated in the form:

$$Pr = \frac{\mu \hat{C}_p}{k_t} = \frac{v}{\alpha} \quad (33)$$

The turbulent Prandtl number, according to Vieser *et al.* (2002), represents an analogy of heat transfer in laminar and turbulent conditions. According to experimental results, for boundary layers formed in regions of solid walls, it is assumed constant and approximately equal to 0.85.

Dimensionless formulation of the $\kappa - \varepsilon$ turbulence model

Applying the nondimensionalization process developed in Equations (7) to (9), we have the dimensionless formulation of the $\kappa - \varepsilon$ turbulence model. Given by:

$$\frac{v_t}{v} = C_\mu \frac{\mathbb{K}^2}{\mathbb{E}} \quad (34)$$

$$\frac{\partial[\bar{V}_j\mathbb{K}]}{\partial X_j} = \frac{\partial}{\partial X_j} \left[\left(1 + \frac{v_t}{\sigma_\kappa} \right) \frac{\partial \mathbb{K}}{\partial X_j} \right] + \frac{v_t}{v} \left(\frac{\partial \bar{V}_i}{\partial X_j} + \frac{\partial \bar{V}_j}{\partial X_i} \right) \frac{\partial \bar{V}_i}{\partial X_j} - \mathbb{E} \quad (35)$$

$$\frac{\partial[\bar{V}_i\mathbb{E}]}{\partial X_i} = \frac{\partial}{\partial X_i} \left[\left(1 + \frac{v_t}{\sigma_\varepsilon} \right) \frac{\partial \mathbb{E}}{\partial X_i} \right] - \mathbb{E} \quad (36)$$

$$+ C_{\varepsilon 1} \frac{v_t}{v} \frac{\mathbb{E}}{\mathbb{K}} \left(\frac{\partial \bar{V}_i}{\partial X_j} + \frac{\partial \bar{V}_j}{\partial X_i} \right) \frac{\partial \bar{V}_i}{\partial X_j} - C_{\varepsilon 2} \frac{\mathbb{E}^2}{\mathbb{K}}$$

Dimensionless formulation of the $\kappa - \omega$ SST turbulence model

Applying again the developed nondimensionalization methodology, now in Equations (10) to (15), we have the dimensionless form of the $\kappa - \omega$ SST turbulence model. Given by:

$$\frac{\partial[\bar{V}_j\mathbb{K}]}{\partial X_j} = \min \left[\frac{v_t}{v} \frac{\partial \bar{V}_i}{\partial X_j} \left(\frac{\partial \bar{V}_i}{\partial X_j} + \frac{\partial \bar{V}_j}{\partial X_i} \right); 10\beta^* \mathbb{K} \mathbb{W} \right] - \beta^* \mathbb{K} \mathbb{W} + \frac{\partial}{\partial X_j} \left[\left(1 + \sigma_\kappa \frac{v_t}{v} \right) \frac{\partial \mathbb{K}}{\partial X_j} \right] \quad (37)$$

$$\frac{\partial[\bar{V}_j\mathbb{W}]}{\partial X_j} = \frac{\alpha^*}{2} \left(\frac{\partial \bar{V}_i}{\partial X_j} + \frac{\partial \bar{V}_j}{\partial X_i} \right)^2 - \beta' \mathbb{W}^2 + \frac{\partial}{\partial X_j} \left[\left(1 + \sigma_\omega \frac{v_t}{v} \right) \frac{\partial \mathbb{W}}{\partial X_j} \right] + 2(1 - F_1) \sigma_{\omega 1} \frac{1}{\mathbb{W}} \frac{\partial \mathbb{K}}{\partial X_j} \frac{\partial \mathbb{W}}{\partial X_j} \quad (38)$$

$$F_1 = \tanh \left\{ \min \left[\max \left(\frac{\sqrt{\mathbb{K}}}{\beta^* \mathbb{W} \mathbb{Y}}, \frac{500}{\mathbb{Y}^2 \mathbb{W}} \right), \frac{4\bar{\rho} \sigma_{\omega 2} \mathbb{K} v^2}{CD_{\kappa\omega} \mathbb{Y}^2 L_p^4} \right] \right\} \quad (39)$$

$$CD_{\kappa\omega} = \max \left(2\bar{\rho} \sigma_{\omega 2} \frac{v^2}{L_p^4} \frac{1}{\mathbb{W}} \frac{\partial \mathbb{K}}{\partial X_j} \frac{\partial \mathbb{W}}{\partial X_j}, 10^{-10} \right) \quad (40)$$

$$F_2 = \tanh \left[\left[\max \left(\frac{2\sqrt{\mathbb{K}}}{\beta^* \mathbb{W} \mathbb{Y}}, \frac{500}{\mathbb{Y}^2 \mathbb{W}} \right) \right]^2 \right] \quad (41)$$

$$\frac{v_t}{v} = \frac{0.3\mathbb{K}}{\max \left(0.3\mathbb{W}; \frac{\sqrt{2}}{2} \left(\frac{\partial \bar{V}_i}{\partial X_j} + \frac{\partial \bar{V}_j}{\partial X_i} \right) F_2 \right)} \quad (42)$$

From all that has been discussed, Equations (3) to (5) are applied for the modeling and dimensionless study of the turbulent natural convection in an isothermal square plate (generic surface geometry and placed in an “infinite” environment). The solution of the mathematical closure problem of turbulence occurs through Equations (34) to (36), for the $\kappa - \varepsilon$ dimensionless turbulence model or through Equations (16) and (37) to (42), for the model of dimensionless turbulence $\kappa - \omega$ SST.

Applicability Discussions

The several characteristics, advantages, and applications of the Theory of Dimensional Analysis and Similarity are widely discussed in the main bibliographical references on the subject. However, despite being a well-established subject in the scientific literature, few bibliographic references – considering publications from the last five years – specifically approach the nondimensionalization methodology of transport equations and turbulence models. In this sense, this work hopes to help reduce these gaps in applications in the study of natural convection.

With the evolution of techniques and methodologies applicable in CFD, which are increasingly used by the academic community and industry, the applications of nondimensionalization combined with numerical artifices of computational simulation in natural convection deserve to be highlighted. As an example, there is the numerical and virtual adjustment of the module and/or the direction of gravity acceleration, used in the construction of numerical simulations in different values of the Rayleigh number and/or with applications for inclined plates; both, without making any geometrical or physical properties changes in the constructed numerical model.

The application of the mentioned numerical artifices requires dimensionless physical-mathematical modeling, with the analysis of the flow numerical results also occurring in a dimensionless form; everything to ensure the physical consistency and validation of numerical results obtained. The works of Verdério Júnior *et al.* (2021) and Silva *et al.* (2021) are strong representative examples of this type of study.

CONCLUSIONS

This work presented a methodology for physical-mathematical modeling of turbulent natural convection problems on an isothermal plate in an "infinite" open environment, with the application of nondimensionalization techniques of transport equations and the $\kappa - \varepsilon$ and $\kappa - \omega$ SST turbulence models. The definition of the dimensionless groups used observed the physical-geometric characteristics of the problem situation and used the terms L_p and v/L_p as dimension and velocity that are characteristic of the studied flow, in that order. In the end, it has been established the dimensionless physical-mathematical model that describes the behavior of the analyzed flow.

The nondimensionalization methodology developed in this work aims to provide, and make available in the current scientific literature, a physical-mathematical model for the most common problem situation for analyzing physical problems involving natural convection. Through minor adaptations in the definition of dimensionless groups, this model can be extrapolated and applied to the study of different forms of convection (forced and mixed).

The vast majority of engineering problems are evaluated through dimensionless parameters. The dimensionless physical-mathematical modeling of the problem evidenced the main dimensionless physical parameters of the influence of the solution, which are the numbers of Grashof, Prandtl and Turbulent Prandtl. However, it is not common to find studies involving dimensionless analyses in the scientific literature for turbulence parameters in numerical solutions (such as turbulence models).

Therefore, the methodology and the dimensionless groups of the turbulence parameters κ , ε , ω and μ_t or ν_t presented in this work will help in the modeling process and studies of the most diverse problem situations in engineering.

Finally, it is important to highlight significant similarities in constitution and presentation between the dimensional and nondimensional forms of the physical-mathematical models treated (with the transport equations and turbulence models used). Such similarities are representative and identify the characteristics of physical-geometric and kinematic similarity of both formulations.

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REFERENCES

- Bejan, A., 2013, *Convection Heat Transfer*, John Wiley & Sons.
- Bergman, T. L. *et al.*, 2014, *Fundamentos da Transferência de Calor e de Massa*, LTC (in Portuguese).
- Bird, R. B., Stewart, W. E. and Lightfoot, E. N., 2004, *Transport Phenomena*, John Wiley & Sons.
- Boussinesq, J., 1897, *Théorie de l'écoulement tourbillonnant et tumultueux des liquides dans les lits rectilignes a grande section*, Gauthier-Villars et fils, Vol. 1. Available in: <http://archive.org/details/thbeoriedelbeco01bousrich>. Access Date: 14 aug. 2021.
- Çengel, Y. A., and Cimbala, J. M., 2015, *Mecânica dos Fluidos: Fundamentos e Aplicações*, AMGH (in Portuguese).
- Çengel, Y. A., and Ghajar, A. J., 2012, *Transferência de Calor e Massa: Uma abordagem prática*, AMGH (in Portuguese).
- Fox, R. W. *et al.*, 2018, *Introdução à Mecânica dos Fluidos*, LTC (in Portuguese).
- Frank, A., Heidemann, W. and Spindler, K. 2019, *Electronic Component Cooling Inside Switch Cabinets: Combined Radiation and Natural Convection Heat Transfer*, Heat Mass Transfer, Vol. 55, pp. 699–709.
- Freile, R. *et al.*, 2021, *Improved natural convection heat transfer correlations for reactor*

cavity cooling systems of high-temperature gas-cooled reactors: From computational fluid dynamics to Pronghorn, Annals of Nuclear Energy, Vol. 163, p. 108547.

Incropera, F. et al., 2008, *Fundamentos da Transferência de Calor e de Massa*, LTC (in Portuguese).

Kitamura, K., Mitsuishi, A., Suzuki, T. and Kimura, F., 2015, *Fluid Flow and Heat Transfer of Natural Convection Adjacent to Upward-Facing, Rectangular Plates of Arbitrary Aspect Ratios*, International Journal of Heat and Mass Transfer, Vol. 89, pp. 320–332.

Liu, L. et al., 2021, *CFD analysis of steam condensation with air in the tubes bundle channel under natural convection conditions*, Annals of Nuclear Energy, Vol. 162, p. 108510.

Menter, F., Kuntz, M. and Langtry, R., 2003, Ten Years of Industrial Experience With the SST Turbulence Model, in: *International Symposium on Turbulence: Heat and Mass Transfer*, Begell House, Antalya, pp. 625-632.

Pope, S. B., 2000, *Turbulent Flows*, England: Cambridge University Press.

Silva, V. R. et al., 2021, *Study and Validation of meshes in turbulent isothermal problems of natural convection in flat plates*, Revista de Engenharia Térmica, Vol. 20, No. 2, p. 33-40. DOI: <http://dx.doi.org/10.5380/reterm.v20i2.81785>.

Verdério Júnior, S. A. et al., 2021, *Physical-numerical parameters in turbulent simulations of natural convection on three-dimensional square plates*, International Journal of Numerical Methods for Heat & Fluid Flow. DOI: <https://doi.org/10.1108/HFF-02-2021-0128>.

Versteeg, H. K. and Malalasekera, W., 2007, *An Introduction to Computational Fluid Dynamics – The Finite Volume Method*, Longman Scientific and Technical.

Vieser, W., Esch, T., and Menter, F., 2002, *Heat transfer predictions using advanced two-equation turbulence models*, CFX Validation Report.

Wang, S. et al., 2021, *Experimental and numerical study on the heat transfer to supercritical water in an inclined smooth tube*, International Journal of Thermal Sciences, Vol. 170, p. 107111.