

NUMERICAL STUDY OF HEAT TRANSFER IN EXTENDED SURFACES WITH MUTUAL RADIATION BETWEEN PARALLEL FINS

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ABSTRACT

The present work shows the influence of the mutual heat transfer on the effectiveness of finned surfaces. Numerical simulations are carried out through a sequence of linear problems, possessing an equivalent minimum principle, that has as its limit the solution of the original problem. The original nonlinear problem is regarded as the limit (which always exists) of a sequence of linear problems like the classical conduction-convection ones. In this work the nonlinear conduction-radiation heat transfer process is considered and simulated by means of a finite difference linear scheme. Such a limit is reached in an easy way by means of standard procedures, allowing the employment of more realistic hypotheses, like some nonlinear boundary conditions, since the mathematical complexities are not a constraint for simulating the elliptic partial differential equation. This work accounts for the the steady state heat transfer process in rigid fins which experiences convective and radiative heat exchange. Some typical results are shown in order to illustrate the methodology. Results have shown both the relevance of the radiation and the importance of the thermal interaction between the fins, so that there is an effective and realistic thermal mapping. Neglecting the thermal interaction can lead to errors of up to 20 percent.

Keywords: nonlinear heat transfer, numerical simulation, sequence of linear problems

NOMENCLATURE

div	denotes the divergence
$grad$	denotes the gradient
h	convection coefficient
k	thermal conductivity
n	the unit outward normal
I	coercive functional
T	absolute temperature
T_∞	environment temperature
\bar{T}	average temperature
T_b	primary surface temperature
d	distance between fins
x, y, z	usual cartesian coordinates

Greek symbols

ε	emissivity factor
δ	thickness of fins
σ	Stefan Boltzman constant
κ_{12}	form factor
α	convergence term
Ω	bounded open set
$\partial\Omega$	the boundary of Ω
β	auxiliar term
\mathcal{U}	term related to sequence of problems
Φ	elements of sequence

INTRODUCTION

The main techniques of heat transfer enhancement are usually those that enhance existing heat transfer elements. A growing demand for engineering projects is related to energy transitions that require more and more efficient heat exchange from/to devices.

Modern times require high performance heat transfer components with progressive weight reduction, volume and cost. Heat transfer on extended surfaces comes to be the study of these components of high performance heat transfer with respect to the most varied parameters and their respective behaviors in the thermal mapping.

This work focuses the heat transfer process in a system of two fins, taking into account for the mutual thermal radiation exchange between them. Mutual radiation has a fundamental role in projects related to photovoltaic panels, aircraft, satellites, thermoelectric generators, thermal treatments of components and in the most different applications industries involving high temperatures and/or rarefied atmospheres. Recent studies have shown the relevance of such considerations in the most diverse engineering applications.

Bjørk et al. (2014) investigated different mechanisms of heat loss in a thermoelectric generator. The model works with non-linear

boundary conditions, including mutual radiation between adjacent surfaces. Alifanov et al. (2016) presented a methodological approach for the thermal mapping of a small spacecraft. The mathematical model of heat transfer considers the external radiant flux field and the influence of mutual radiation between spacecraft surfaces. Cheema et al. (2016) investigated numerically the heat transfer in tubular furnaces considering mutual radiation properties among their constituents. Tubular furnaces are used in the annealing of crystalline structures of metals at high temperatures and the determination of the temperature distribution inside a tubular furnace is an important coefficient for efficiency in the heat treatment.

The present work numerically investigates the performance of coupled heat transfer in longitudinal fins arrays considering mutual radiation between points on the fins. Conduction-radiation heat transfer process is an inherently non-linear phenomenon in which the coupling on the boundary of the body is mathematically represented by a nonlinear relationship between the absolute temperature and its outside normal derivative. The unknown is the temperature distribution. The solution of the problem is achieved by the limit of a sequence whose elements are obtained from the minimization of a quadratic functional.

NUMERICAL FORMULATION

Enhancement heat transfer techniques have been the subject of multiple studies for several geometries (Kundu and Bhanja, 2010), (Lemouedda et al., 2011) and (Torabi and Aziz, 2012). Experimental procedures are more costly and may present different results from standards, due to the difficulty of working with low temperature differences and/or high coefficients of heat transfer. Analytical difficulties may arise due to particularities of the solution of non-linear thermal phenomena. Therefore, numerical simulation techniques are required to obtain effective and realistic results.

Mathematical Model

Steady-state analysis become realistic in most problems concerning extended surface applications, some specific cases such as high-speed aircraft applications and automatic control devices require care with transient term (Kraus et al., 2002).

The steady-state heat equation without internal heat generation with constant thermal conductivity and non-linear boundary conditions for a single fin is given by

$$\frac{\partial}{\partial x} \left(\frac{\partial \bar{T}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial \bar{T}}{\partial y} \right)$$

$$- \frac{2}{k\delta} \left[h(\bar{T} - T_\infty) + \varepsilon\sigma|\bar{T}|^3 \cdot \bar{T} \right] = 0 \quad \text{in } \Omega_1 \quad (1)$$

in a configuration represented by $0 < x < L$, $0 < y < b$ e $0 < z < \delta$, in which \bar{T} is the mean temperature, δ is the thickness of fins, k is the thermal conductivity, h is the convection coefficient, T_∞ is the environment temperature, ε is the emissivity factor and σ is the Stefan-Boltzmann constant.

Since the thickness of the fin is assumed to be very small, the partial differential equation is integrated over the variable z . The average value for the temperature is defined as

$$\bar{T} = \frac{1}{\delta} \int T dz \quad (2)$$

For two fins in steady state, as shown in Fig. 1, without internal heat generation with constant thermal conductivity and nonlinear boundary conditions, taking into account mutuality in the emission of radiation between adjacent fins and combined effect of mutual and environmental radiation, the heat equation becomes

$$\frac{\partial}{\partial x} \left(\frac{\partial \bar{T}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial \bar{T}}{\partial y} \right) - \frac{2}{k\delta} \left[h(\bar{T} - T_\infty) + \varepsilon\sigma|\bar{T}|^3 \cdot \bar{T} - \frac{1}{2} \int \sigma|\bar{T}|^3 \cdot \bar{T} \kappa_{12} dS \right] = 0 \quad (3)$$

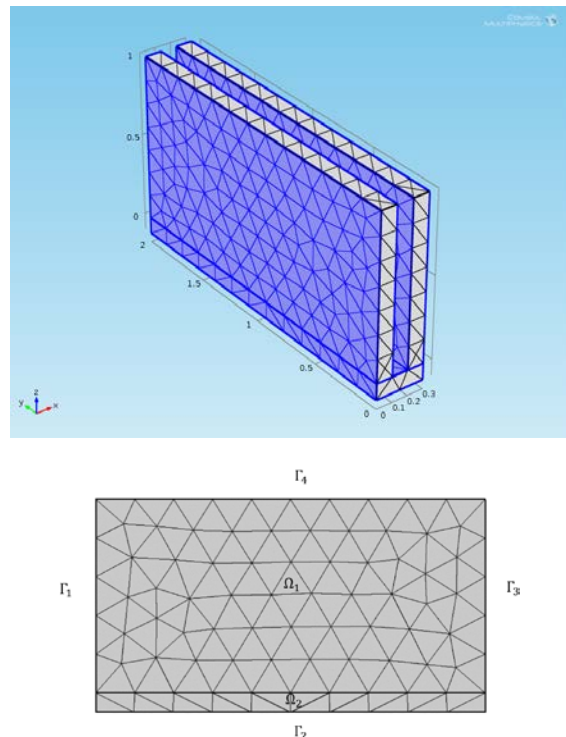


Figure 1. Dual fins and boundary conditions.

For dimension L much greater than b, we have for $0 < y < b$

$$\frac{d^2 \bar{T}}{dy^2} - \frac{1}{k\delta} \left[\begin{array}{l} 2h(\bar{T} - T_\infty) + 2\sigma|\bar{T}|^3 \bar{T} \\ - \int_0^b \sigma|\bar{T}(\xi)|^3 \bar{T}(\xi) \frac{d^2}{2((y-\xi)^2 + d^2)^{3/2}} \end{array} \right] = 0 \quad (4)$$

Eq. (5) is a portion referring to the emission of radiation between fins, as shown

$$\int_0^b \sigma|\bar{T}(\xi)|^3 \bar{T}(\xi) \frac{d^2}{2((y-\xi)^2 + d^2)^{3/2}} d\xi \quad (5)$$

where κ_{12} is the form factor and d the distance between the fins. The boundary conditions (b.c.) are homogenous of Neumann in Γ_1, Γ_3 e Γ_4 and Dirichlet b.c. in Γ_2 . Equations (1), (3), (4) e (5)

employ the term $|\bar{T}|^3 \bar{T}$ in place of term T^4 to guarantee operator coercivity of infinite dimension, thus preserving the physical structure of the phenomenon of heat transfer.

Sequence of Problems

In order to ensure the existence of a minimum, solution for Eq. (1) and Eq. (3) is necessary and sufficient to show that I is continuous, convex and coercive functional, where α is the convergence term, β is the auxiliar term and v is the related term with sequence of the problems.

The solution of problem in Eq. (1) may be reached as the limit of a sequence whose elements are obtained from the minimization of a quadratic functional.

$$I[v] = \frac{1}{2} \int_0^H \int_0^Z \left[\left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] dx dy + \int_0^H \int_0^Z \left[\alpha v^2 - 2\beta v \right] dx dy \quad (6)$$

The first variation of Eq. (6) describes the variational formulation of the physical problem, which must be represented mathematically by

minimizing the coercive functional. The existence of the minimum principle provides a simple and precise tool for numerical simulation of the phenomenon of heat transfer.

In other words:

$$\omega = \lim_{i \rightarrow \infty} \Phi_i \quad (7)$$

in which the elements of the sequence $[\Phi_0, \Phi_1, \Phi_2, \dots, \Phi_i]$ are obtained by

$$\text{div}(\text{grad}\Phi_{i+1}) = \alpha\Phi_{i+1} - \beta_i \quad \text{in } \Omega_1 \quad (8)$$

and

$$-(\text{grad}\Phi_{i+1})n = 0 \quad \text{on } \partial\Omega_1 \quad (9)$$

the β auxiliar term:

$$\beta_i = \alpha\Phi_{i-1} - \left(\sigma|\Phi_{i-1}|^3 \Phi_{i-1} - h(\Phi_{i-1} - T_\infty) \right) \quad \text{for } i = 0, 1, 2, \dots \quad (10)$$

where α is sufficiently large positive constant and $\Phi_0 \equiv 0$. This constant is evaluated from an a priori estimate for the upper bound of the solution and ensures a bounded and nondecreasing sequence $[\Phi_0, \Phi_1, \Phi_2, \dots, \Phi_i]$.

The problem proposed in this work originally has Robin boundary conditions, but the proposed methodology simplifies the convergence solution mode, imposing Neumann boundary condition without physical sense, but being an efficient mathematical tool.

In other words, the minimum of the functional is reached for the field $v = \Phi_{i+1}$ which satisfies Eq. (1). It is to noticed that Φ_i is a prescribed function, when we look for the minimum of the functional. The tools employed for solving linear heat transfer problems. The constant α must large enough to ensure that

$$\Phi_{i+1} \geq \Phi_i, \quad \text{in everywhere} \quad (11)$$

The temperature is given by:

$$T = \lim_{i \rightarrow \infty} \Phi_i \quad \text{in } \bar{\Omega} \quad (12)$$

The elements of the sequence $[\Phi_0, \Phi_1, \Phi_2, \dots, \Phi_i]$ are obtained from the minimization of a continuous, convex and coercive functional, this assures the existence and the uniqueness of each element and

provides an efficient way for obtaining them. In addition, it is assured the existence and the uniqueness of the sequence $[\beta_0, \beta_1, \beta_2, \dots, \beta_i]$.

The sequences $[\Phi_0, \Phi_1, \Phi_2, \dots, \Phi_i]$ and $[\beta_0, \beta_1, \beta_2, \dots, \beta_i]$ are, for each i , non-decreasing, provides only the solution with physical sense. This fact allows us to conclude that the solution is the unique non-negative, since, from classical thermodynamics, the absolute temperature must be a non-negative value real field, we conclude that Eq. (1) and Eq. (8) are thermodynamically equivalent. The algorithm associated to the minimization of the functional I provide an efficient procedure for simulating the considered energy transfer phenomena. In addition, this procedure provides only the (desired) solution with thermodynamical sense.

RESULTS AND DISCUSSION

Recent studies have reported numerical-experimental problems in multi-finned heat sinks with linear boundary conditions using multiphysics software (Diani et al., 2013), (Chen et al., 2016) and (Yang et al., 2016). Others obtained numerical solutions in FDM, FEM and FVM in the most diverse matrices neglecting radiation nonlinear boundary conditions (Tiari et al., 2015), (Li et al., 2016), (Senapati et al., 2016) and (Kim et al., 2016). Dogonchi et al. (2016) verified behavior of the steady state single rectangular fin with nonlinear boundary conditions and thermal conductivity varying linearly with temperature.

This work introduces the idea of the sequence of linear problems to find a solution of the steady-state problem in fins with non-linear boundary conditions whose thermal conductivity is constant. Moreover, the mathematical analysis is based on the modifications of Murray-Gardner’s hypotheses so that realistic problems are solved. Such procedures include the effects of: non-zero heat sink temperature and mutual radiation between parallel fins.

The distribution of temperatures in the fins will be observed in 4 different cases: single fin with constant thermal conductivity without radiation, single fin with constant thermal conductivity with radiation, dual fins with constant thermal conductivity without radiation, dual fins with constant thermal conductivity with radiation.

Numerical Convergence

Tab.1 illustrates the numerical convergence process, presents $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6$ and Φ_∞ obtained with three different values of $\alpha (\alpha = \alpha_1 = 1.10^1 W / m^2 K)$, $(\alpha = \alpha_2 = \alpha_1^2)$ and $(\alpha = \alpha_3 = \alpha_1^3)$.

Table 1. Numerical convergence verification.

Φ_i	$\alpha = \alpha_1$	$\alpha = \alpha_2$	$\alpha = \alpha_3$
$i = 10$.5945	.5941	.5940
$i = 100$	5.2373	5.2245	5.0802
$i = 500$	9.8306	9.8270	9.7566
$i = 1000$	10.0053	10.0052	10.0021
$i = 1200$	10.0081	10.0076	10.0059
$i = 1387$	10.0081	10.0081	10.0079
$i \rightarrow \infty$	10.0081	10.0081	10.0081

Single and Dual Fins with and without Radiation

Figure 2 was obtained for fins with height $h = 10mm$ and thickness $\delta = 10mm$. The meshes consisted of 50 x 50 nodes under surrounding temperature of $T_\infty = 300K$ and primary surface temperature of $T_b = 500K$.

Analyzing the results obtained and comparing them in Fig. 2, is evident the relevance of the phenomenon of radiation in extended surfaces, being clear that realistic experiments should not neglect this phenomenon.

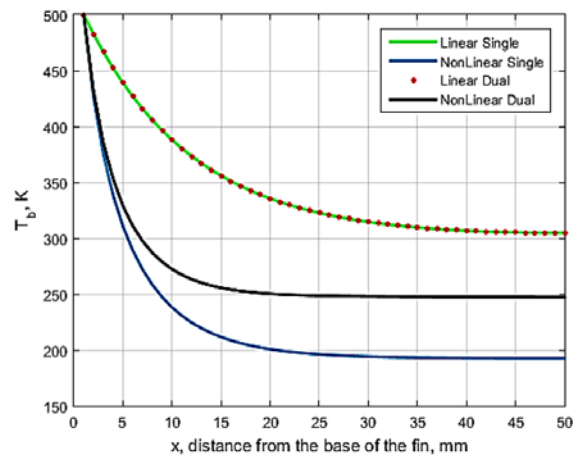


Figure 2. Single and dual fins with and without radiation.

Mutual Radiation Effect between Parallel Fins

Tables 2 and 3 denote expressive relevance of mutual radiation between radiating elements for dual parallel fins with convective and radiative boundary conditions. At the top of the fins is observed percentage differences of 22.39%.

Table 2. Mutual radiation effect for dual fins on the first 5 nodes.

	1	2	3	4	5
Without mutuality	500	426.14	375.31	338.33	310.41
With mutuality	500	431.71	385.62	352.94	328.99
Percentage difference	0	1.29	2.67	4.14	5.65

Table 3. Mutual radiation effect for dual fins on the last 5 nodes.

	1	2	3	4	5
Without mutuality	191.92	191.91	191.91	191.90	191.90
With mutuality	247.32	247.30	247.27	247.27	247.27
Percentage difference	22.40	22.40	22.39	22.39	22.39

Dimensionless Parameters

This item will take a brief analysis of item 3.2 taking into account dimensionless parameters in order to generalize the verified observations.

For N number of nodes, k thermal conductivity, C_1 and C_2 convection and radiation constants respectively, we have that the variation of the dimensionless temperature and the dimensionless position along the fin will have particularities according to the parameter showed in Fig. 3, that is, the relation between height and distance between the fins. Fig. 3 shows that when this relationship tends to infinity the interaction between the fins is despicable, for analytical aims there is no interaction between fins, accordingly considering single fin. U_1 , U_2 and U_∞ being the local temperatures and ambient temperature. In Fig. 3 shows the relationship between dual and single fin temperatures with dimensionless position, attempting to the maximum point in this curve.

Being U_1 , U_2 and U_∞ respective local temperatures and environment temperature. The Fig. 3 denotes a relation between dual and single temperatures fins varying with the dimensionless position, it is observed that there is a maximum point for such analysis.

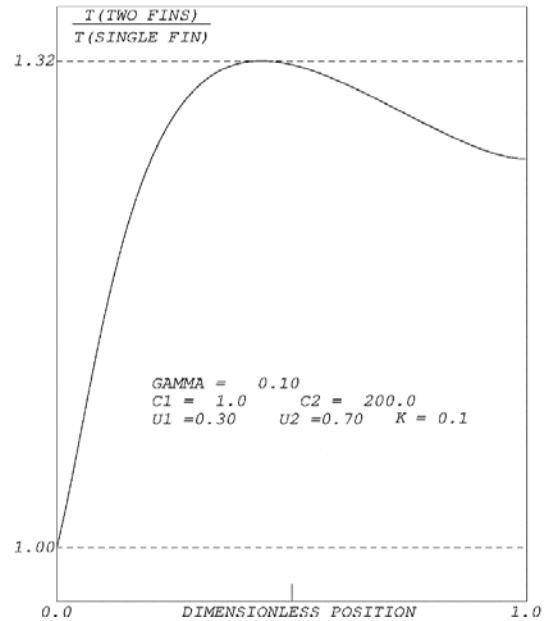


Figure 3. Comparison and relation of dimensionless temperatures with position.

CONCLUSIONS

The knowledge of the actual thermal mapping conditions plays a key role for an optimized design. A considerable methodology was presented in the present study, using theoretical, analytical and numerical treatment for finned surfaces. Results have shown both the relevance of the radiation and the heat mutuality so that there is an effective and realistic mapping.

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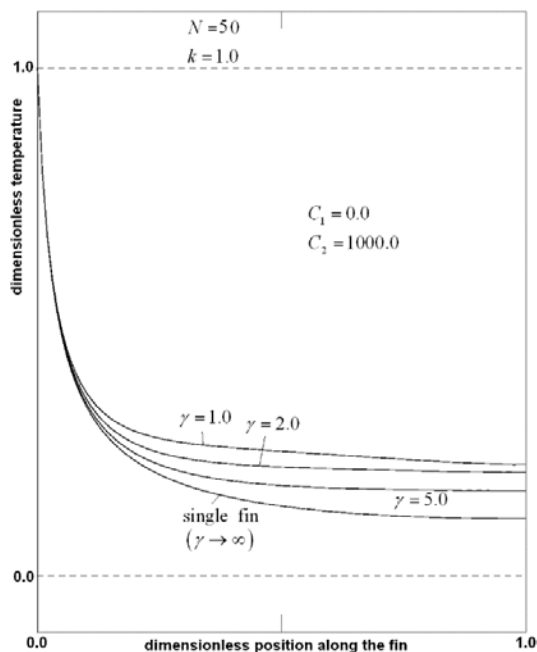
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