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GEOMETRIC OPTIMIZATION OF "+"-SHAPED CAVITY USING CONSTRUCTAL THEORY

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ABSTRACT

In this work it is applied the Constructal Theory for the study of the geometry of an "+"-shaped isothermal cavity inserted in a conductive solid body. Main goal is to minimize the maximum temperature in the solid. The total volume of the solid and the total volume of the cavity are kept fixed while the dimensions of the cavity geometry vary according to constraints and degrees of freedom defined by the Constructal Design. The solid body has internal heat generation and its external surfaces are insulated. Cavity walls are isothermal with constant temperature T_{min}. Obtained results indicate that the optimal performance of "+"-shape cavity is 37.2% better that the optimal performance of "C"-shape cavity and 10.8% better than the "T"-shaped cavity for the same thermal conditions.

Keywords: isothermal cavity, PDETool, geometric optimization, heat conduction

NOMENCLATURE

A	area, m^2
A_c	cavity area, m^2
H	height, m
H_0	cavity region height, m
L	length, m
L_0	cavity region length, m
k	thermal conductivity, W m^{-1} K^{-1}
q	heat current, W
q'''	volumetric rate of heat generation, W m^{-3}
c_p	specific heat, W $kg^{-1}K^{-1}$
t	time, s
T	temperature, K
x, y	spatial coordinates, m

Greek symbols

θ	dimensionless temperature
ρ	specific mass, kg·m-^{3}

Subscripts

min	minimum
max	maximum

Superscripts

~	dimensionless variables

INTRODUCTION

Constructal Theory states the fundamental idea that everything that moves, whether animate or inanimate, is a flow system. All flow systems generate shape and structure over time in order to facilitate this movement through a landscape full of resistance (e.g. friction) (Bejan, 2000; Bejan and Lorente, 2008; Bejan and Lorente, 2013 and Bejan and Zane, 2012).

In the last years, engineering problems related to the heat transfer area have received great attention. Many of these problems focus on explaining how the internal geometry of solid surfaces influences the behavior of heat transfer. They seek, in addition to understanding, to improve their performance and search for new geometries, due to the importance for various applications, such as heat exchanger, internal combustion, electric motors and thermal conductors.

Several works show the great interest that many researchers are giving to the Constructal Theory and how it is being applied in problems of optimization of the shape of flow systems that generate geometries and structures. Works such as Biserni et al. (2007), Gonzales et al. (2015), Link et al. (2013), Lorenzini et al. (2014), Lorenzini et al. (2012), Lorenzini et al. (2014), Lorenzini et al. (2011) and Lorenzini and Rocha (2009) studied isothermal cavities with different shapes intruded into solids with internal heat generation aiming to increase heat transfer, performance.

In this work, a cavity cooling problem of a steady state heat conduction solid with internal heat generation is solved with the finite element method.

Geometry is 2D with constant physical properties (density and conductivity). Constructal Theory and Exhaustive Search are used to minimizes the maximum temperature in the solid domain. The cavity has a "+"-shaped form with constant area. Main goal of current solution is to determine best cavity geometry.

The Matlab PDETool is used to perform the different simulations. This tool is a commercial software based in the Finite Element Method (FEM) for solution of partial differential equations and it is used here to solve the heat diffusion equation for achievement of thermal field in the solid domain. In addition, it allows the construction of the computational domain from basic forms (rectangles, ellipses and polygons), generate and refine meshes and define the boundary conditions, being a useful platform to approach to this type of problem (MATLAB, 2010).

MATHEMATICAL AND NUMERICAL MODELING

In this work it, is presented a steady-state conduction heat transfer problem in which there is a two-dimensional solid (plate) with constant thermal conductivity (k) and uniform heat generation at a volumetric rate (q'''). The solid has completely isolated boundaries and the heat generated is removed by the "+"-shaped cavity walls which are maintained at a minimum temperature (T_{min}). Fig. 1 (a) shows plate, cavity and boundary conditions.

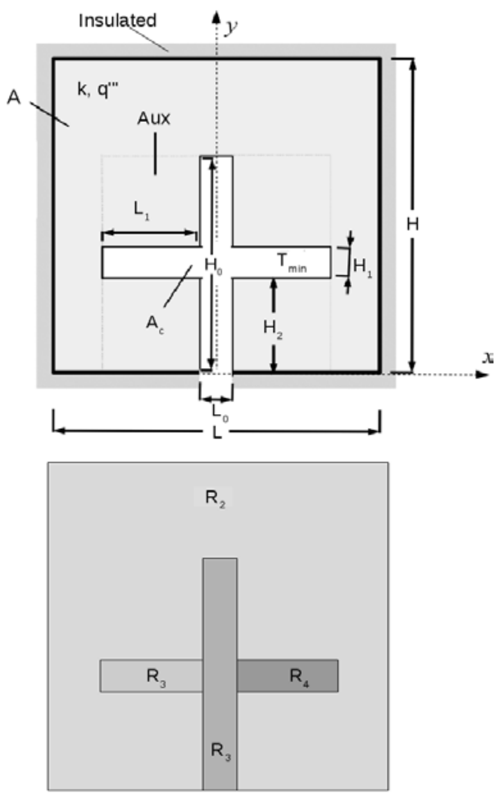


Figure 1. Problem and cavity description.

Plate has height H and length L. The "+"-shaped cavity can be defined as the composition of the regions R_2, R_3 and R_4 as shown in Fig. 1 (b), where a region R_2 has height H_0 and length L_0 and the regions R_3 and R_4 having equal dimensions being H_1 the height and L_1 the length. According the Constructal Theory, geometry optimization can be subject to two constraints. The first being the total area

$$A = H \cdot L \tag{1}$$

and the second corresponds to the area of the cavity which is given by

$$A_c = H_0 \cdot L_0 + 2 \cdot H_1 \cdot L_1 \tag{2}$$

The constraints imposed by Eqs. (1) and (2) are inserted into the formulation by defining a dimensionless variable, $\emptyset$, such that

$$\emptyset = \frac{A_c}{A} \tag{3}$$

In addition, H_2 is defined as the lower formation point of the regions R_3 and R_4.

The maximum temperature that occurs in the computational domain depends on the isothermal cavity geometry. Main goal is to optimize cavity geometry in order to minimize the dimensionless thermal resistance. The analysis that allows calculating the dimensionless thermal resistance as a function of the geometry consists of numerically solving the heat conduction equation given by

$$k\frac{\partial^2\theta}{\partial\tilde{x}^2} + k\frac{\partial^2\theta}{\partial\tilde{y}^2} + 1 = 0 \tag{4}$$

where the dimensionless variables are:

$$\theta = \frac{T - T_{min}}{q'' \, A / k} \tag{5}$$

$$\tilde{x}, \tilde{y}, \tilde{L}_0, \tilde{L}_1, \tilde{H}_0, \tilde{H}_1, \tilde{H}_2, \tilde{H}, \tilde{L} = \frac{x, y, L_{0,} L_{1,} H_{0,} H_{1,} H_{2,} H, L}{A^{1/2}} \tag{6}$$

Dimensionless thermal resistance is defined by:

$$\theta_{max} = \frac{T_{max} - T_{min}}{q''' \, A / k} \tag{7}$$

The mesh will be constructed with non-uniform triangular elements in two dimensions, having the mesh for each geometric configuration between 900 and 45000 elements. Mesh refinement is made and validated until the criterion of grid independence is satisfied. This criterion is defined by the equation:

$$\left(\theta_{max}^{j}-\theta_{max}^{j+1}\right)/\theta_{max}^{j} < 1\cdot 10^{-4} \tag{8}$$

Table 1 shows an example of how grid independence is calculated.

Table 1. Grid independence test (H/L=1, Ø=0.13, $H_0/L_0 = 14.26$, $H_1/L_1 = 0.33$, $H_2 = 0.445$).

Number of elements	θ_{max}^{j}	$(\theta_{max}^{j}-\theta_{max}^{j+1})/\theta_{max}^{j}$
754	0.0683	9.54×10^{-3}
3016	0.0690	3.56×10^{-3}
12064	0.0694	1.41×10^{-3}
48256	0.0695	5.69×10^{-4}
193024	0.066	

Conduction problem addressed needs the following boundary conditions:

- Dirichlet boundary conditions - constant surface temperature:

$$T_{min} = const \tag{9}$$

- Neumann boundary conditions-constant thermal flux on the surface (a particular case when the surface is adiabatic or isolated as in the current problem where q" is considered equal to 0):

$$-k\frac{\partial T}{\partial n} = q'' \tag{10}$$

Thus, by defining the external surface of the domain as adiabatic, the boundary conditions are given by the equations:

$$\frac{\partial\theta}{\partial\tilde{x}} = 0 \text{ in } \tilde{x} = -\tilde{L}/2 \text{ or } \tilde{x} = \tilde{L}/2 \text{ and } 0 \le \tilde{y} \le \tilde{H} \tag{11}$$

$$\frac{\partial\theta}{\partial\tilde{y}} = 0 \text{ in } \tilde{y} = 0 \text{ and } -\tilde{L}/2 \le \tilde{x} \le -\tilde{L}_0/2 \text{ or } \tilde{L}_0/2 \le \tilde{x} \le \tilde{L}/2 \tag{12}$$

$$\frac{\partial\theta}{\partial\tilde{y}} = 0 \text{ in } \tilde{y} = \tilde{H} \text{ and } -\tilde{L}/2 \le \tilde{x} \le \tilde{L}/2 \tag{13}$$

Now, being

$$x_b = \frac{\tilde{L}_0}{2} + \tilde{L}_0$$

and

$$y_b = \tilde{H}_2 + \tilde{H}_1$$

the boundary conditions for the cavity surface are given by

$$\theta = 0 \text{ in } \tilde{x} = -\tilde{L}_0/2 \text{ or } \tilde{x} = \tilde{L}_0/2 \text{ and } 0 \le \tilde{y} \le \tilde{H}_2 \text{ or } y_b \le \tilde{y} \le \tilde{H} \tag{1}$$

$$\theta = 0 \text{ in } -x_b \le \tilde{x} \le -\tilde{L}_0/2 \text{ or } \tilde{L}_0/2 \le \tilde{x} \le x_b \text{ and } \tilde{y} = \tilde{H}_2 \text{ or } \tilde{y} = y_b \tag{2}$$

$$\theta = 0 \text{ in } \tilde{x} = -x_b \text{ or } \tilde{x} = x_b \text{ and } \tilde{H}_2 \le \tilde{y} \le y_b \text{ or } \tilde{y} = y_b \tag{3}$$

$$\theta = 0 \text{ in } -\tilde{L}_0/2 \le \tilde{x} \le \tilde{L}_0/2 \text{ and } \tilde{y} = \tilde{H} \tag{4}$$

Function defined by Eq. (7) can be numerically determined by solving Eq. (4) to the temperature field in each assumed configuration. Then θ_{max} is calculated in order to study its dependence on each geometric configuration.

RESULTS

For the development of this work three degrees of freedom were considered: dimensionless values directly linked to the geometry evolution, and two constraints defined by the Constructal Design theory. The degrees of freedom considered for the "+"-shaped cavity study and the definition of their appearance are: H_0/L_0, H_1/L_1 and H_2. The defined constraints are the total area A and cavity area A_c. In addition, the dimensions of H_0, L_0, H_1, L_1 and H_2 may vary, but will be restricted to maximum and minimum values, in accordance with the degree of freedom.

The search for the geometry that minimizes resistance to heat flow follows three steps. In the first one, geometry optimization is sought by varying the relation H_0/L_0 and keeping fixed the other degrees of freedom. In the second step, the relation H_1/L_1 is varied and the other parameters are kept fixed using the new value of H_0/L_0. In the third and last stage, the influence of H_2 on the behavior of the temperature inside the solid is studied. For this last stage, the other degrees of freedom are kept fixed to the values obtained in the previous steps. Initially, behavior analysis of θ_{max} are made for different cavity configurations where $\emptyset = 0.13$ and $H/L = 1.0$.

In the first step simulation, it is considered the variation of ratio H_0/L_0, that defines the shape and size of the region R_2 of the cavity and are kept fixed $H_1/L_1 = 0.33$ ($H_1 = 0.1$ and $L_1 = 0.3$) and $H_2 = (H_0 - H_1)/2$. Constraint tested for the variation of the degrees of freedom are: $0.18 \le H_0 < 1$ and $0.07 \le L_1 < 0.4$. In Tab. 2, it is possible to see the values of θ_{max} for some values of H_0/L_0 evaluated and the amount of grid elements that previously satisfies the criterion of mesh independence.

Table 2. Optimization of H_0/L_0 ($\emptyset = 0.13$, $H/L = 1.0$, $H_1/L_1 = 0.33$ and $H_2 = (H_0 - H_1)/2$).

H_0/L_0	Number of elements	θ_{max}
0.44	1416	0.2739
7.00	76288	0.1085
14.26	7488	0.066

Fig. 2 shows the behavior of the maximum temperature for the different simulated configuration. It is possible to observe that the increase of the ratio H_0/L_0 leads to a decrease of maximum temperature in the domain.

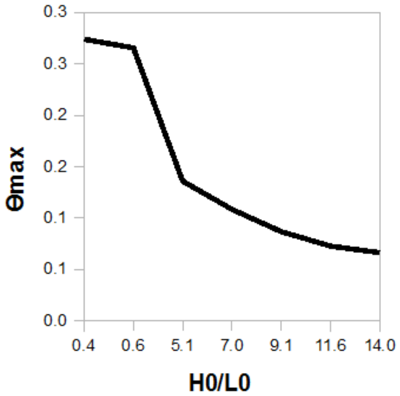


Figure 2. Effect of the ratio H_0/L_0 over the maximum temperature (θ_{max}).

In Fig. 3, the configurations and temperature field for data presented in the Tab. 2 are shown.

For the tested configurations, $H_0/L_0 = 14.26$ presented the lowest maximum temperature. In this configuration $H_0 = 0.999$, $L_0 = 0.07$ and θ_{max}=0.066.

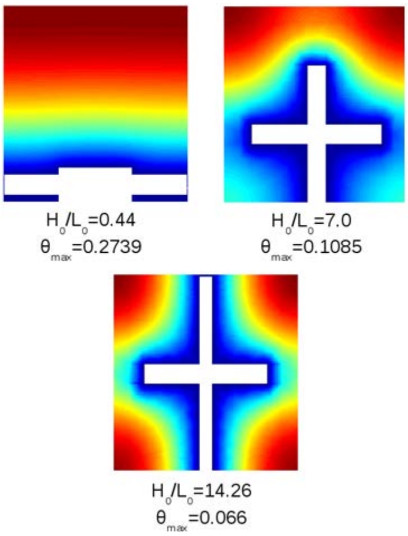


Figure 3. Geometric configurations of different values of H_0/L_0 ratio ($\emptyset = 0.13$, $H/L = 1.0$, $H_1/L_1 = 0.33$ and $H_2 = (H_0 - H_1)/2$).

In the second stage, it is performed the variation of the H_1/L_1 degree of freedom which defines the shape and size of regions R_3 and R_4 of the cavity, simultaneously. For this case, the value for the optimized H_0/L_0 ratio in the previous stage was used. Thus, the following parameters are fixed for the simulations: $H_0/L_0 = 14.26$ and $H_2 = (H_0 - H_1)/2$. The restrictions for the variation of tested degrees of freedom were: $0.065 \leq H < 0.998$ and $0.03 \leq L < 0.464$.

Simulation was performed for different ratios between H_1 and L_1 and some results are presented in Tab. 3.

Table 3. Results obtained for different values of the ratios H_1/L_1 ($\emptyset = 0.13$, $H/L = 1.0$, $H_0/L_0 = 14.26$ and $H_2 = (H_0 - H_1)/2$).

H_1/L_1	Number of elements	θ_{max}
0.14	6760	0.0611
0.75	49152	0.0747
33.2	21824	0.0877

In addition, temperature behavior is presented in Fig. 4, allowing to observe that as lower is the ratio H_1/L_1 lower is the flux resistance. Thus, the geometry which presented the best results can be seen in Fig. 5, where $H_1/L_1 = 0.14$ with $H_1 = 0.065$, $L_1 = 0.465$ and θ_{max}=0.0611.

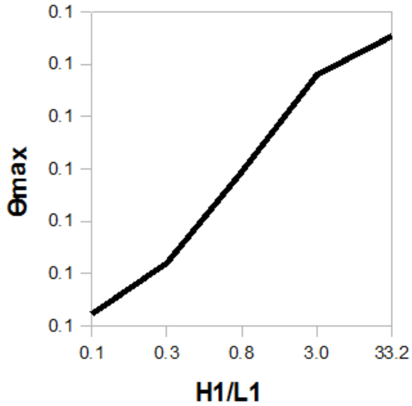
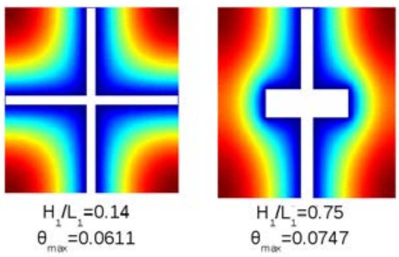


Figure 4. Behavior of θmax for different values of H_1/L_1 ($\emptyset = 0.13$, $H/L = 1.0$, $H_0/L_0 = 14.26$ and $H_2 = (H_0 - H_1)/2$).



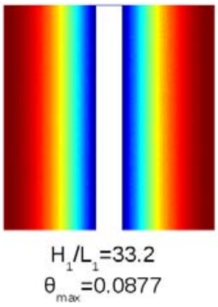


Figure 5. Some geometric configurations for different values H_1/L_1 ($\emptyset = 0.13$, $H/L = 1.0$, $H_0/L_0 = 14.26$ and $H_2 = (H_0 - H_1)/2$).

Third stage consists in evaluating the geometry considering the variation of the degree freedom H_2. For this case, it was kept fixed: $H_0/L_0 = 14.26$. Third degree of freedom, H_2, was made to vary from 0 to 0.93.

Results after having solved the heat diffusion equation for different values of H_2 are shown in Tab. 4 and dimensionless thermal resistance variation is plotted in Fig. 6.

Table 4. Results obtained for different values of H_2 ($H_0/L_0 = 14.26$ and $H_1/L_1 = 0.14$).

H_2	**Number of element**	θ_{max}
0.001	1416	0.0916
0.2	6764	0.0839
0.467	**27040**	**0.0611**
0.6	7328	0.0748
0.93	5096	0.0915

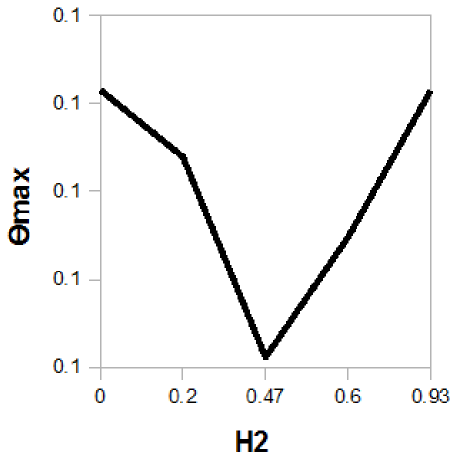


Figure 6. Behavior of θ_{max} for different values of H_2 ($H_0/L_0 = 14.26$ and $H_1/L_1 = 0.14$).

Observing Table 4, it is clear that the configuration which leads to minimal value of θ_{max} is obtained when H_2 is such that R_3 and R_4 as positioned at the center of R_2 region, i.e., when H_2 is close to $(H_0 - H_1)/2$. Thermal resistance for different values of $\emptyset$ are presented in Fig. 7

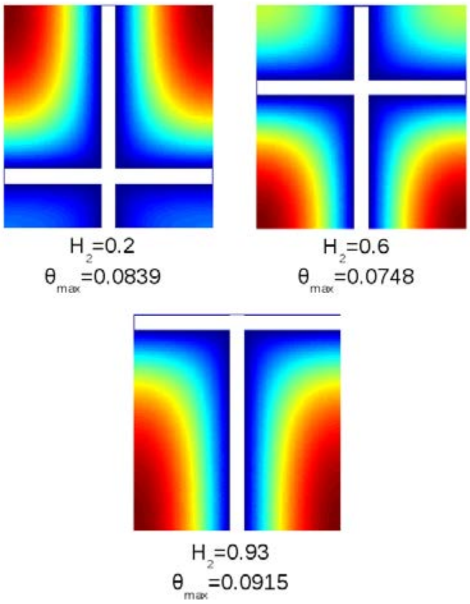


Figure 7. Some geometric configurations for different values of H_2 ($H_0/L_0 = 14.26$ and $H_1/L_1 = 0.14$).

Therefore, by applying results obtained in the search for optimal cavity geometry for $\emptyset = 0.13$, geometries are constructed for different ratios between the total and cavity areas. That is, the geometries are constructed by maintaining the H_0/L_0 maximum value, H_1/L_1 minimum value, and $H_2 = (H_0 - H_1)/2$. Results of the best geometries for $\emptyset = 0.1$, $\emptyset = 0.2$ and $\emptyset = 0.3$ are shown in the Fig. 8.

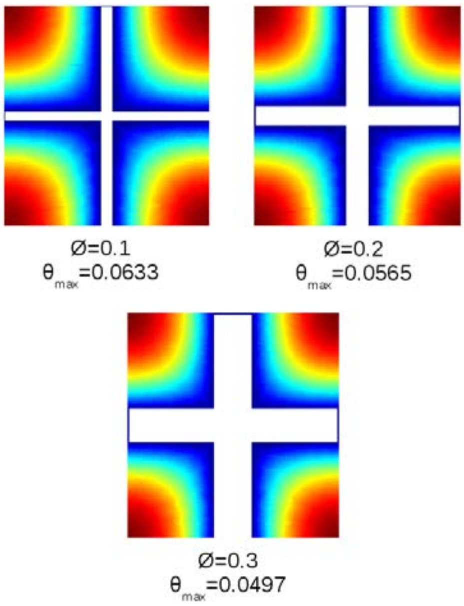


Figure 8. Best geometric configurations for $\emptyset = 0.1$, $\emptyset = 0.2$ and $\emptyset = 0.3$ with $H/L = 1.0$.

Results of this work for the $\emptyset = 0.1$ and optimized C-shaped, T-shaped, H-shaped and X-shaped cavities were compared in Tab. 5. Results indicate that the performance of the optimum "+"-shape cavity is 37.2% better than the C-shape cavity and 10.8% better than of the T-shape cavity.

Table 5. Comparison of cavities ($H/L = 1.0$ and $\emptyset = 0.1$).

	θ_{max}
C-shape cavity (Biserni, Rocha and Bejan, 2004)	0.1008
T-shape cavity (Lorenzini et al. 2014)	0.0710
H-shape cavity (Biserni et al. 2007)	0.0245
X-shape cavity (Link et al. 2013)	0.0395
"+"-shape cavity	0.0633

However, it has a performance of 61.3% lower than that of the H-shape cavity and 37.6% lower than that of the X-shape cavity.

CONCLUSIONS

In this work, a heat transfer problem was addressed in which an isothermal "+"-shaped cavity is located inside a conductive plate with heat generation. During the study, simulations were performed using the PDETool of Matlab software to study different configurations of geometry cavity, focusing on optimization, in order to minimize resistance to heat flux.

Constructal Design assists in defining the constraints and degrees of freedom of the problem geometry. The plate is insulated and with internal and a uniform heat generation. In addition, the total and cavity areas are kept constant.

Simulations were performed by varying the ratio between the geometry dimensions, optimizing one degree of freedom in each step of the process, being called "optimal" those configurations that have a lower resistance to heat flow. Results indicate that when $\emptyset = 0.1$, the "+"-shape cavity has a θ_{max}=0.0633, i.e, it is 37.2% better than the optimum C-shape cavity and 10.8% better than the T-shape cavity.

In addition, the use of the Constructal Theory allows to better explain the geometric configuration that offers a better thermal performance of the system.

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