

TWO-PHASE FRICTION FACTOR IN GAS-LIQUID PIPE FLOW

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ABSTRACT

An improved friction factor prediction model for two-phase gas-liquid pipe flow is proposed. The model is based on a previous no-slip formulation where a mixture Reynolds number was defined. In this study, the mixture Reynolds number is modified by introducing slip-ratio information through the inclusion of void-fraction and flow-pattern dependent models. An experimental database reconstituted from the available literature and new frictional pressure-drop data for air-water horizontal flow in an I.D. 0.0204m pipe are also presented. The full database considers several different flow conditions for horizontal two-phase flow of refrigerants and air-water mixtures. It was compared to predictions of models from the literature as well as the new proposed model. We found that the proposed and Müller-Steinhagen-and-Heck methods provide better agreement for the current experimental database. It is shown that the inclusion of void-fraction information on the previous mixture Reynolds definition improves the friction-factor prediction.

Keywords: mixture Reynolds number, friction factor, two-phase flow, gas-liquid flow, phenomenological modeling

NOMENCLATURE

A	area, m ²
C_1, C_2	proposed method constants
d	diameter, m
dH	hydraulic diameter, m
dP/dL	pressure drop gradient
e	statistical parameter, %
e/d	relative roughness
g	gravitational constant, m/s ²
J	superficial velocity, m/s
L_{test}	length of the test line, m
P	perimeter, m
P_{system}	system pressure, kPa
Re	Reynolds number
Re_M	Mixture Reynolds number by Shannak (2008)
V	actual velocity, m/s

Greek symbols

α	void fraction
β	homogeneous void fraction
ξ	friction factor
η	statistical parameter, %
μ	viscosity, Pa-s
ρ	density, kg/m ³

Subscripts

exp	experimental
F	frictional
$-FP$	flow pattern
G	gas
H	Homogeneous
I	Interface
L	Liquid
$pred$	Predicted
$-\alpha$	void fraction

INTRODUCTION

Pressure-drop prediction plays a relevant and important role in operation and design of pipeline systems. The Darcy method is the most popular for single-phase flow and its friction-factor estimation has been the focus of significant research efforts for over a century. It is known that for smooth pipes the friction factor is strongly related to the Reynolds number, but for pipes having significant roughness it also depends on the relative roughness (Colebrook and White, 1937). In the case of two-phase flow, some theoretical and experimental approaches have been developed to predict pressure drop. A review of the state-of-the-art can be found in

Cravino *et al.* (2009). Many of the models are based on the no-slip homogeneous theory. According to García and García (2009), the models show good agreement with experimental data when an appropriate selection of the mixture viscosity and density equations is done. Following this approach, Shannak (2008) presents a no-slip model to predict the friction factor, and, consequently, two-phase frictional pressure drop. Shannak's formulation is original because it proposed a new concept by redefining the mixture Reynolds number. Recently, Ortiz-Vidal *et al.* (2012) showed that although the method by Shannak predicts the frictional pressure drop in horizontal air-water flow more accurately, when compared with classical methods (Chisholm, 1983, 1967; Dukler *et al.*, 1964; Lockhart and Martinelli, 1949), the method excludes relevant two-phase flow information because of the homogeneous hypotheses.

In the present study, the mixture Reynolds number defined by Shannak is modified by introducing two-phase flow phenomenology through the inclusion of void-fraction and flow-pattern parameters. This approach avoids the need for adopting the no-slip homogeneous assumption of the original method. Therefore, an extension of the no-slip model is proposed. The performance of the proposed model is evaluated through comparison with several reported models and new and available two-phase flow data from the literature.

PROPOSED METHOD

Shannak (2008) presents a methodology to estimate the friction factor for two-phase gas-liquid flow. The model is based on the redefinition of the mixture Reynolds number (Re_M) as the ratio of the sum of inertial forces of the two phases over the sum of viscous forces of the phases,

$$Re_M = \frac{\sum F_L}{\sum F_V} = \frac{\rho_L J_L^2 d^2 + \rho_G J_G^2 d^2}{\mu_L J_L d + \mu_G J_G d} \quad (1)$$

No-slip considerations were adopted to evaluate Re_M . Shannak's model applies the homogeneous model (Wallis, 1969), using Re_M in the friction-factor correlations obtained directly from single-phase-flow hydraulics, for example, correlation by Chen (1979),

$$\xi_{pred} = \left[-2 \log \left(\frac{1}{3.7065} \left(\frac{e}{d} \right) \right. \right. \\ \left. \left. - \frac{5.0452}{Re} \log \left(\frac{1}{2.8257} \left(\frac{e}{d} \right)^{1.1098} \right) + \frac{5.8506}{(Re)^{0.8981}} \right) \right]^{-2} \quad (2)$$

where e/d represents the relative roughness. The proposed model is based on the approach of Shannak. However, further phenomenological information about the flow is included in the mixture Reynolds number. Inertial and viscous forces are evaluated as functions of the actual velocities and equivalent diameters of each phase,

$$Re_{M-\alpha} = \frac{\rho_L V_L^2 d_L^2 + \rho_G V_G^2 d_G^2}{\mu_L V_L d_L + \mu_G V_G d_G} \quad (3)$$

where $V_L (= J_L/(1-\alpha))$ and $V_G (= J_G/\alpha)$ represent the actual velocity of liquid and gas, respectively. The separate-cylinder model is applied to determine the equivalent diameters (Wallis, 1969),

$$d_L^2 = (1-\alpha)d^2 \\ d_G^2 = \alpha d^2 \quad (4)$$

In the particular case of separated flow, such as stratified and annular flow, equivalent diameters could be replaced by hydraulic diameters from geometrical considerations. Thus, the proposed method acquires a second version specifically for separated flow,

$$Re_{M-FP} = \frac{\rho_L V_L^2 d_{HL}^2 + \rho_G V_G^2 d_{HG}^2}{\mu_L V_L d_{HL} + \mu_G V_G d_{HG}} \quad (5)$$

where

$$d_{HL}^2 = C_1 (1-\alpha)d^2, \quad C_1 = (1-\alpha)C_{1*}^2 \\ d_{HG}^2 = C_2 \alpha d^2, \quad C_2 = \alpha C_{2*}^2 \quad (6)$$

C_{1*} and C_{2*} are separated-flow parameters and are functions of the interfacial and phases perimeters (see Table 1). Table 2 presents the geometrical expressions for calculating the perimeters. In the case of stratified flow these perimeters were obtained considering a gas-liquid flat interface and are function of the dimensionless phase level (h_e). For other flow patterns C_1 and C_2 are assumed to be equal to 1.0 according to the equivalent diameter approach. Flow pattern information is needed to evaluate Re_{M-FP} . The proposed models $Re_{M-\alpha}$ and Re_{M-FP} include phenomenological information through the inclusion of void fraction and flow-pattern parameters.

Table 1. Parameters C_{1*} and C_{2*} of Eq. 5.

	C_{1*}	C_{2*}
If $V_G > V_L$		
stratified/annular	$\pi d / P_L$	$\pi d / (P_G + P_L)$
else ($V_G < V_L$)		
stratified	$\pi d / (P_L + P_I)$	$\pi d / P_G$

Table 2. Flow geometry for separated flow patterns stratified.

If $\alpha \geq 0.5$,

$$P_L = \arccos(1-2h_e)d, P_G = \pi d - P_L$$

$$P_I = d\sqrt{1-(1-2h_e)^2}$$

solve $A_L = (1-\alpha)A = A_{phase}$ to give h_e else ($\alpha < 0.5$),

$$P_G = \arccos(1-2h_e)d, P_L = \pi d - P_G$$

$$P_I = d\sqrt{1-(1-2h_e)^2}$$

solve $A_G = \alpha A = A_{phase}$ to give h_e

where,

$$A_{phase} = \left[\begin{array}{l} \arccos(1-2h_e) \\ -(1-2h_e)\sqrt{1-(1-2h_e)^2} \end{array} \right] \frac{d^2}{4}$$

annular

$$P_L = \pi d, P_G = 0, P_I = \pi d\sqrt{\alpha}$$

EXPERIMENTAL WORK

Experimental facilities

The experimental work was conducted at the Fluid-Structure Interaction Laboratory of the BWC/AECL/NSERC Industrial Research Chair, Polytechnique Montreal. Figure 1 illustrates schematically the two-phase test loop, where the main components and measurement instruments are designated by letters and numbers, respectively. The experimental setup was specifically constructed to study flow-induced vibration subject to two-phase pipe flow, according to Ortiz-Vidal (2012).

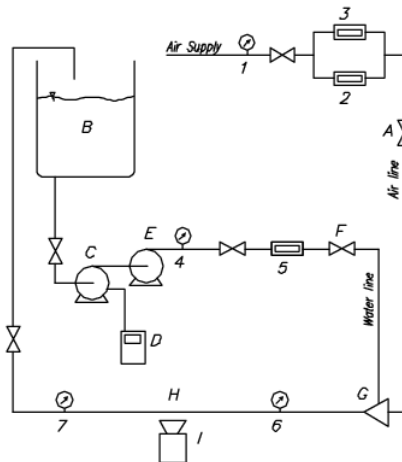


Figure 1. Schematic representation of the two-phase test loop. The letters and numbers represent the main components and measurement instruments, respectively.

Compressed air at steady pressure, controlled by the regulator 1, is supplied into the system. The air

flow rate is measured and regulated by the flowmeters 2 and 3 and the valve A. The air flowmeters 2 (0-75 slpm) and 3 (0-1000 slpm) have 1% and 1.5% FS accuracy, respectively. From an independent line, water at standard ambient temperature and pressure is pumped from the reservoir B by the centrifugal pumps C and E. The liquid flow rate is set by a frequency inverter acting on the pump C and it is measured upstream by the flowmeter 5 with 0.25% RD accuracy. Air and water are mixed by means of a fitting type Y (G, Fig. 1). The two-phase mixture flows, in several flow patterns, through the test section (H, Fig. 1). The fluids are separated by gravity in the atmospheric tank B. The test section consists of a 3/4-in schedule 40 commercial clear PVC pipe with inner diameter $d = 20.4$ mm and a span of $L_{test} = 90 d$.

In order to measure inlet and outlet pressure, pressure transducers (0-50 psi) with 0.05% FS accuracy (6 and 7, Fig. 1) are installed L_{test} apart from each other. These collected signals were employed to calculate pressure drop and the effective pressure at the midspan position (P_{system}). The latter was used to adjust the desired air flow rate. A hydrodynamic development length of 1760 mm ($\approx 86 d$), between the mixer exit and the first pressure tap (6, Fig. 1) was considered. This fact ensures a fully development flow for all tested conditions, according to the approach proposed by Ortiz-Vidal (2012). Images of the two-phase flow patterns were taken using the high-speed video camera system I located at the test section midspan position, *i.e.* 131 d away from the mixer exit. A program developed in LabVIEW™ was used for acquiring and processing the signals. The sampling rate of 5000 Hz was chosen to ensure the accuracy of the sensors signals and to avoid the aliasing effect.

Experimental conditions

A set of 32 two-phase air-water flow conditions were investigated. The mixture velocity, $J (= J_L + J_G)$, and the homogeneous (no-slip) void fraction, $\beta (= J_G / (J_L + J_G))$, were used to specify the experimental points. Every flow condition was set to the midspan position of the test section (H, Fig. 1). Hence the readings of the air flow meters were corrected by averaging the pressure values downstream and upstream (locations 7 and 6, Fig. 1). The nominal values of mixture velocities ranged from 0.5 to 25 m/s and the homogeneous void fractions values were $\beta = 10\%$, 25%, 50%, 75% and 95%. Figure 2 shows the test conditions in terms of gas and liquid superficial velocities, J_G and J_L , respectively.

Not only well-defined bubbly, dispersed and slug flow patterns, but also transition flow patterns were observed. For example, in the case of bubbly-dispersed flow transition, there are bubbles accompanied by small gas bubbles flowing on the top of the test sections; then, the presence of small

bubbles increased and bubbles almost disappeared. Gradually the small bubbles occupied the pipe cross-sectional area until it was completely filled, *e.g.* fully dispersed flow pattern. Similar bubbly-slug and slug-dispersed flow pattern transitions were observed. In order to classify all tested conditions the flow pattern definitions of Shoham (2006) was used, taking into account the topological configurations predominantly observed. Figure 2 shows the results, where a good agreement between experiments and the flow map of Mandhane *et al.* (1974) is observed.

Table 3. Experimental results of the present study.

JL [m/s]	JG [m/s]	P_{system} [kPa]	dP/dL_F [kPa/m]	Pressure drop uncertainty (\pm)
0.45	0.05	104.9	0.188	7.03%
0.91	0.10	107.1	0.714	4.37%
1.81	0.20	112.9	2.442	2.25%
2.70	0.29	123.5	5.033	1.18%
3.60	0.40	138.5	8.189	0.70%
4.51	0.50	157.4	12.095	0.46%
6.30	0.67	211.4	21.486	0.23%
0.38	0.12	105.0	0.188	6.94%
0.75	0.25	107.0	0.676	4.51%
1.50	0.49	113.4	2.329	2.13%
2.25	0.73	122.8	4.770	1.21%
3.00	0.99	137.2	7.888	0.72%
3.75	1.23	156.3	11.081	0.47%
5.25	1.73	214.2	17.993	0.22%
0.26	0.25	104.5	0.150	7.86%
0.50	0.50	105.6	0.376	5.85%
1.01	0.99	110.3	1.615	2.85%
1.49	1.50	117.6	3.268	1.58%
2.50	2.49	144.2	7.926	0.59%
3.50	3.55	181.9	12.997	0.31%
5.00	4.97	258.9	21.448	0.16%
0.24	0.75	104.3	0.263	8.39%
0.50	1.53	106.5	0.826	4.94%
1.25	3.80	121.2	4.207	1.30%
2.50	7.47	170.3	11.720	0.37%
3.75	11.07	276.1	24.153	0.14%
4.36	12.97	353.4	31.290	0.10%
0.24	4.75	104.5	0.826	8.47%
0.50	9.56	111.4	2.667	2.58%
0.75	14.27	125.3	5.597	1.09%
1.00	18.93	146.1	9.992	0.58%
1.25	23.67	181.1	14.762	0.32%

Table 3 shows the experimental superficial velocities, system pressure and frictional pressure gradient. The last column represents the combined uncertainty of the pressure transducers (6 and 7,

Figure 1). Relatively high uncertainty values are observed for low pressure-gradient values, as expected since the accuracy is calculated having as reference the pressure transducer's full scale.

EXPERIMENTAL DATABASE

Manageable two-phase-flow experimental data is quite scanty in the open literature. Most studies present experimental data in figures with swarms of points, which makes the identification of data points an impossible task. For this study, a reasonably broad pressure-drop database was collected from Kanizawa (2011), Moreno-Quibén (2005) and Shannak (2008). These literature data and the present experimental data are used to evaluate the proposed method. In total there are 412 experimental points considering several different diameters, working fluids and flow conditions. Table 4 shows the characteristics of the chosen database. DB, BB, IN, SL, SS, SW stand for dispersed bubbly, bubbly, intermittent, slug, stratified smooth and stratified wavy flow, respectively. In order to illustrate the full Table 4 database, the experimental flow map of Mandhane *et al.* (1974) was used, as shown in Fig. 2. It can be observed that the flow pattern of Mandhane *et al.* (1974) has a good agreement with Kanizawa (2011) and our own data. In the case of Moreno I and II databases, that author recommended the map by Wojtan (2004) to predict flow patterns Moreno-Quibén (2005). In this study, when comparing the flow map of Mandhane *et al.* (1974) and Wojtan (2004) an acceptable agreement was also found.

Experimental friction-factor data for two-phase flow was obtained from the pressure-drop database described in Table 4. The Darcy expression,

$$\xi_{exp} = \left(\frac{2d}{\rho_H J^2} \right) \left(\frac{dP}{dL} \right)_F \quad (7)$$

was used, where $(dP/dL)_F$ is the experimental frictional pressure-drop gradient and $\rho_H (= (1-\beta)\rho_L + \beta\rho_G)$ the mixture density assuming a no-slip homogeneous model (Wallis, 1969). The homogeneous mixture Reynolds number is used to correlate the Darcy friction factor. It is expressed by

$$Re_H = \frac{\rho_H J d}{\mu_H} \quad (8)$$

where $\mu_H (= (1-\beta)\mu_L + \beta\mu_G)$ is the mixture viscosity (Dukler *et al.*, 1964). The experimental friction factor, as a function of the homogeneous mixture Reynolds number, is presented in Fig. 3.

To calculate the experimental friction-factor, the temperature values reported by those authors of Table 4 were used. For the present data, a reference room temperature value of 20°C, corresponding to the

outlet fluid temperature of the test loop, was considered. We found that the air temperature has a very low influence on friction-factor calculation. A sensitivity analysis was carried out to evaluate the effect of temperature on mixture density. Simulations

with air temperatures of 15°C and 25°C have shown that the deviation of friction factor values from those calculated at reference room temperature are negligible. The deviations calculated were up to 0.21%. Figure 4 illustrates these results.

Table 4. Characteristics of database.

Source	Kanizawa (2011)	Moreno I (Moreno-Quibén, 2005)	Moreno II (Moreno-Quibén, 2005)	Shannak (2008)	Present data
Fluid considered	R134a	R22	R134a*	Air-Water (AW)	Air-Water (AW)
Total data points	134	96	71	79	32
Diameter (mm)	15.8	13.8	13.8	52.5	20.4
Flow Patterns (FP)	SW,SL,SS,IN	not reported	not reported	not reported	DB,SL,EB
Liquid superficial velocity (J_L)	0.003 - 0.19	0.003 - 0.319	0.005 - 0.23	0.062 - 0.681	0.245 - 6.303
Gas superficial velocity (J_G)	0.164 - 7.031	1.259 - 21.66	0.357 - 16.49	0.477 - 34.56	0.054 - 23.67

* diabatic flow

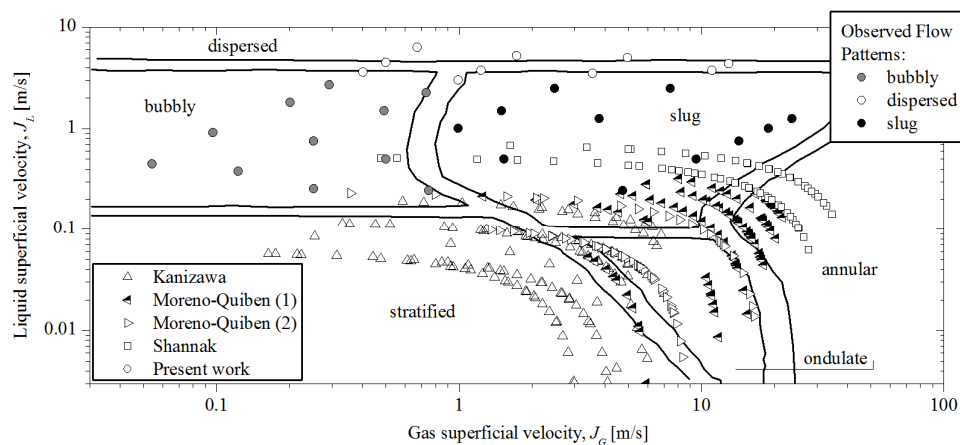


Figure 2. Table 4 experimental database on Mandhane *et al.* (1974)'s flow map. For the present work data, the grey, white and black circles (\circ) represent the bubbly, dispersed and slug flow patterns, according to experimental observations, respectively.

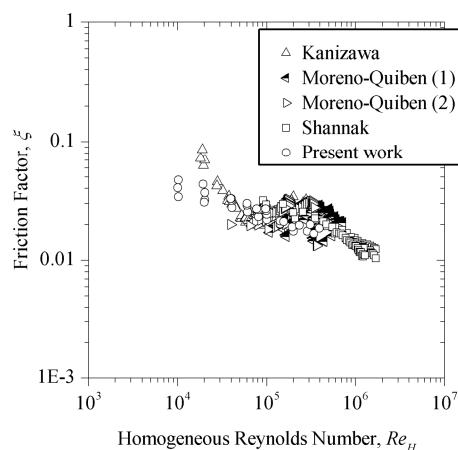


Figure 3. Friction-factor versus homogeneous Reynolds number, refer to Table 4.

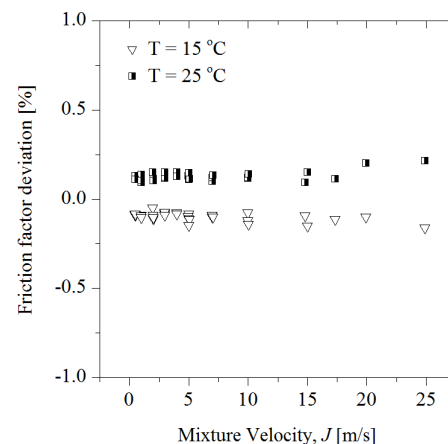


Figure 4. Friction-factor deviation *versus* mixture velocity. The deviations were calculated from a reference temperature-value of 20°C.

METHOD EVALUATION

The accuracy of the proposed method was tested with the experimental database shown in Fig. 3 and compared with the predictions of the following selected methods: Shannak (2008), no-slip homogeneous (Dukler *et al.*, 1964), Müller-Steinhagen and Heck (1986) and García *et al.* (2007). The comparisons were at the same flow conditions compiled in Table 4. Two statistical parameters e and η , representing the average absolute percent error and the percentage of data predicted within $\pm 30\%$, respectively, were used.

$$e = \frac{1}{n} \sum_{i=1}^n \left| \frac{\xi_{\text{pred}} - \xi_{\text{exp}}}{\xi_{\text{exp}}} \right| \times 100\% \quad (9)$$

The predictions of the proposed models, $Re_{M-\alpha}$ and Re_{M-FP} , were obtained using, respectively, Eq. (3) and Eq. (5) and the friction factor correlation of Chen (1979), Eq. (2). Notice that the proposed method is based on the inclusion of the void fraction as a parameter that contains two-phase flow information. Therefore, the correct estimation of void fraction is relevant. Two void-fraction correlations, Armand-Massina and Rouhani-Axelson, were tested (Woldeamayrat and Ghajar, 2007). In the latter, a variation valid for flows of refrigerants was also used Wojtan (2004). In the specific case of the proposed Re_{M-FP} method, flow pattern information was needed, Table 1. The flow maps of Wojtan (2004) and Mandhane *et al.* (1974) were used for Moreno I and II (Moreno-Quibén, 2005) and Shannak (2008) databases, respectively. Flow-pattern information for each experimental condition was available and thus employed in Kanizawa's and the present databases.

The models of Shannak and no-slip homogeneous were applied by substituting Re by Re_M (Eq. (1)) and Re_H (Eq. (8)), respectively, in the friction-factor correlation of Chen, Eq. (2). The friction factor according to Müller-Steinhagen and Heck was obtained indirectly. First, the empirical method proposed by those authors was used to calculate the frictional pressure drop; then, the friction factor was obtained as described in the previous section, Eq. (7). The model of García *et al.* (2007), referred as FFIUC in the original study, was performed using the parameters proposed by those authors. The parameters were obtained from the fitting of a comprehensive experimental database, mainly air-water flow conditions. To estimate the holdup they recommend the utilization of the holdup correlations of García *et al.* (2005). In this study, the so called theoretical correlation TMC was employed. At this point it is important to emphasize two things. First, García *et al.*'s method originally provides predictions for the Fanning friction factor. We converted it to Darcy friction factor, ensuring consistency with the present study where the latter is

used. Second, in the abovementioned holdup study other fitted correlations are also presented (García *et al.*, 2005). However, when they are applied for the flow conditions presented in Fig. 2, a significant percentage of the holdup predictions were above one, which violates the law of conservation of mass. Therefore, those correlations were not used and only the correlation TMC was tested.

Table 5 shows the statistical evaluation of the selected prediction methods for the experimental database (Table 4). One can see a slight difference between friction factors predicted by the proposed method when using either the Armand-Massina or Rouhani-Axelson void-fraction correlations. These differences were less than 0.6% and 1.5% for the statistical parameters e and η , respectively. It should be pointed out that both Armand-Massina and Rouhani-Axelson correlations are accurate (Woldeamayrat and Ghajar, 2007). The agreement between the predictions of the proposed methods and the friction-factor database was fairly good (Table 5), except for Moreno II who ran their experiments under diabatic conditions. The poor predictions for diabatic flow of refrigerant suggest low accuracy in the estimations of the momentum (acceleration) pressure-drop component. This could be inferred from the void-fraction variation along the diabatic test section, according to Moreno-Quibén (2005). The proposed Re_{M-FP} model provides the best prediction for Kanizawa (2011) and the present databases, with percent error $e = 18.6\%$ and percent prediction $\eta = 85.1\%$ and $e = 15.6\%$ and $\eta = 100\%$, and the second best performance for Shannak (2008) and Moreno I databases, with $e = 7.3\%$ and $\eta = 96.2\%$ and $e = 24.0\%$ and $\eta = 70.8\%$, respectively. In the diabatic flow case of Moreno II database, as cited above, a rather poor prediction was observed ($e = 26.9\%$ and $\eta = 54.9\%$). For this database, Müller-Steinhagen and Heck (1986) provides the best prediction with $e = 17.1\%$ and $\eta = 83.1\%$. Moreover, the other selected methods had worse predictions than the proposed methods.

With respect to the present data, the proposed models $Re_{M-\alpha}$ and Re_{M-FP} had the best predictions, as cited above. The values of $e = 15.6\%$ and $\eta = 100\%$ were the same for both, as expected since no separated flow was observed. The selected models also had a good agreement between experiments and predictions; sorted by accuracy they were García *et al.* (2007) ($e = 16.1\%$ and $\eta = 96.9\%$), Shannak (2008) ($e = 17.6\%$ and $\eta = 90.6\%$), Müller-Steinhagen and Heck (1986) ($e = 19.3\%$ and $\eta = 84.4\%$) and no-slip homogeneous ($e = 21.5\%$ and $\eta = 81.3\%$). Although García *et al.* (2007) had the second best performance for the present database, its predictions for the other databases were worse than the no-slip homogeneous method. The poor predictions for Kanizawa (2011), Moreno I and II (Moreno-Quibén, 2005) databases could be because García *et al.* (2007) correlation was not fitted for

refrigerant two-phase flow. There is not an evident explanation of why García *et al.* (2007) also poorly predicts the air-water database of Shannak (2008). The range of experimental conditions of Shannak (2008) database are included on the database used by García *et al.* (2007) for fitting their own correlation. It might be simply a problem of extrapolation of García *et al.* (2007)'s results.

Table 5 also shows the statistical evaluation of the selected prediction methods classified by fluid and flow type. "Refrigerant" includes the diabatic and adiabatic flow conditions of refrigerants, *i.e.* Kanizawa (2011) and Moreno I and II databases. Kanizawa (2011) and Moreno I databases compose "Refrigerant Adiabatic". "AW" includes air-water flow conditions, *i.e.* Shannak (2008) and present databases. "Refrigerant Adiabatic & AW" considers the databases for "Refrigerant Adiabatic" and "AW". Finally, "Total" represents the results for the full Table 4 database.

It can be observed that Müller-Steinhagen and Heck (1986) predicts reliably the two-phase friction

factor for "Refrigerant". On the other hand, its predictions for "AW" are rather poor in comparison with the other methods. Müller-Steinhagen and Heck (1986) and both proposed-model versions had the best predictions for "Refrigerant Adiabatic", where diabatic flow is not considered. In the case of "AW", the proposed methods and Shannak (2008) predicted accurately these database. According to the previous description, and sorted by accuracy, proposed methods, Shannak (2008) and Müller-Steinhagen and Heck (1986) had better predictions for "Refrigerant Adiabatic & AW". "Total" (last column of Table 5) presents Müller-Steinhagen and Heck (1986) and both proposed models as the most accurate ones. In general, García *et al.* (2007) has the worst predictions. It was also found that, on average, the proposed Re_{M-FP} model, which considers flow-pattern information, does not offer significant improvements. Therefore, the proposed Re_{M-a} model has the advantage of being simpler to use.

Table 5. Statistical evaluation of the selected prediction methods for the experimental database (Table 4).

Correlation / data source (fluid considered)	Kanizawa (R134a)		Moreno I (R22)		Moreno II (R134a*)		Shannak (AW)		Present Data (AW)		Refrigerant		Refrigerant Adiabatic		AW		Refrigerant Adiabatic and AW		Total	
Total data points	134		96		71		79		32		301		230		111		341		412	
Statistical Parameters (%)	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η	<i>e</i>	η
Proposed-method- Re_{M-a} - Rouhani-Axelsson	18.7	84.3	24.0	70.8	26.9	54.9	10.4	96.2	15.6	100	22.3	73.1	20.9	78.7	11.9	97.3	18.0	84.8	19.5	79.6
Proposed-method- Re_{M-a} - Armand-Massina	18.9	82.8	24.2	70.8	27.1	54.9	9.9	96.2	16.2	100	22.5	72.4	21.1	77.8	11.7	97.3	18.0	84.2	19.6	79.1
Proposed-method- Re_{M-FP} - Rouhani-Axelsson	18.6	85.1	25.0	67.7	27.8	52.1	7.3	96.2	15.6	100	22.8	71.8	21.3	77.8	9.7	97.3	17.5	84.2	19.3	78.6
Proposed-method- Re_{M-FP} - Armand-Massina	18.8	83.6	25.1	67.7	27.7	53.5	7.2	96.2	16.2	100	22.9	71.4	21.4	77.0	9.8	97.3	17.6	83.6	19.3	78.4
Shannak 2008	20.3	79.1	25.8	66.7	30.1	46.5	6.2	98.7	17.6	90.6	24.4	67.4	22.6	73.9	9.5	96.4	18.3	81.2	20.4	75.2
No-slip Homogeneous	19.1	82.1	28.5	54.2	31.3	38.0	14.9	79.7	21.5	81.3	25.0	62.8	23.0	70.4	16.8	80.2	21.0	73.6	22.8	67.5
Müller-Steinhagen and Heck (1986)	21.4	73.1	14.0	90.6	17.1	83.1	22.6	74.7	19.3	84.4	18.0	81.1	18.3	80.4	21.7	77.5	19.4	79.5	19.0	80.1
García <i>et al.</i> (2007) FFIUC-TMC	36.1	40.3	41.9	37.5	43.9	19.7	35.5	41.8	16.1	96.9	39.8	34.6	38.5	39.1	29.9	57.7	35.7	45.2	37.1	40.8

* diabatic flow

CONCLUSIONS

A method to predict the friction factor for two-phase flow is presented. The model is based on the inclusion of void fraction in the mixture Reynolds number proposed by Shannak (2008). The void-fraction parameter introduces relevant two-phase flow information in the previous no-slip method. Two versions of the new model were proposed, depending on whether the flow pattern is known or unknown. Friction-factor predictions of the proposed model, Shannak (2008), no-slip homogeneous (Dukler *et al.*, 1964), Müller-Steinhagen and

Heck (1986) and García *et al.* (2007) were compared with an available database of air-water and refrigerant diabatic and adiabatic two-phase flow from the open literature. Acceptable results are obtained with the proposed methods, Shannak (2008) and Müller-Steinhagen and Heck (1986). The latter provided the best prediction for refrigerant diabatic flow. On average, the proposed model where the flow pattern is known does not offer significant improvements. Therefore, the version of the proposed model where the flow pattern is unknown is recommended because it has the advantage of being simpler to use. García *et al.* (2007) and no-slip

homogeneous methods had the worse accuracies. The results presented here indicate that the inclusion of void-fraction information in the mixture Reynolds number improves the predictions of friction factor.

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