HIGH-ORDER FINITE DIFFERENCE METHOD APPLIED TO THE SOLUTION OF THE THREE-DIMENSIONAL HEAT TRANSFER EQUATION AND TO THE STUDY OF HEAT EXCHANGERS

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ABSTRACT

Numerical experiments for four test problems are carried out to demonstrate the performance of the present method and to compare it with the others classical methods. The numerical solutions obtained are compared with the analytical solution as well as the results by other numerical schemes with emphasis on the application involving heat exchange in a rectangular channel. It can be easily seen that the proposed method is simple to implement and very efficient..

Keywords: high-order finite difference, numerical solution, heat transfer equation, heat exchangers

NOMENCLATURE

k_x,k_y, k_z thermal conductivityin the *x*,*y*,*z*-directions, respectively
c_p heat capacity
t time
T temperature

Greek symbols

ρ mass density

INTRODUCTION

Problems involving the advection-diffusion equation have important applications to fluid dynamics as well as many other branches of science and engineering.Because the analytical solution of these equations containing complex initial and boundary conditions are very difficult, many authors have used various numerical techniques for the solution of this equation as the finite element methods(Zienkiewicz et al., 2013), the finite volume methods(Malalasekera and Versteeg, 2007), the shooting method (Roberts and Shipman, 1972),the graphical methods (Welty et al., 2001),the modified domain decomposition method (Wazwaz, 2001), the Cole-Hopf transformation (Fletcher, 1983), the radial basis function collocation method (Islam et al., 2012), the differential transform method (Liu and Hou, 2011), the differential quadrature method (Mittal and Jiwari 2009), the Adomian decomposition method (Zhu et al., 2010) and the finite difference methods (Thomas, 1995, Morton and Mayers, 1994, Forsythe and Wasow, 2013) used in this work, are other approaches for solve the advection-diffusion equation.

In recent years, there exist a lot of studies devoted to the numerical approximation to the three-dimensional advection-diffusion equations and its applications using the high-order finite difference method for solution. For example, (Dehghan, 2004) developed several second order fully explicit (unconditionally stable) and fully implicit (conditionally stable) difference schemes with constants coefficients. (Thongmoon et al., 2007) used the finite difference method to solve the three-dimensional advection-diffusion that describes a mathematical model for transport of a pollutant in a street tunnel. Pollutant dispersal patterns within the tunnel were calculated and numerical results for several different pollutant source configurations were presented and discussed. (Prieto et al., 2011) used the application of the generalized finite difference method to solve the advection-diffusion equation by the explicit method and studied the convergence of the method and the truncation error over irregular grids. The example presented numerical example shows that a decrease in the value of the time step, always below the stability limits, leads to a decrease in the global error. (Ge et al., 2013) developed an exponential high order compact alternating direction implicit method with fourth-order in space, second order in time and unconditionally stable and solved three numerical problems to demonstrate the high accuracy and efficiency and to show its superiority over the classical Douglas-Gunn ADI scheme and the Karaa's high order ADI scheme.

Among the range of applications, simulations of rectangular cooling channels, such as happens in cold storage, have often been discussed in recent works (Smale et al., 2006; Hao and Ju, 2011) and will be operated in the Application 4 of this work.

GOVERNING EQUATIONS: TEMPORAL AND SPATIAL DISCRETIZATION

A In this work, we propose a solution by the high-order finite difference method for the three-dimensional advection-diffusion equation which is given by.

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}-\frac{k_x}{\rho c_p}\frac{\partial^2 T}{\partial x^2}-\frac{k_y}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{k_z}{\rho c_p}\frac{\partial^2 T}{\partial z^2}=0 \tag{1}$$

considering k_x, k_y e k_z as thermal conductivity in the x,y,z-directions, respectively, ρ as mass density and c_p as heat capacity.

Rearranging the Eq. (1), taking $\alpha_x=\frac{k_x}{\rho c_p}$, $\alpha_y=\frac{k_y}{\rho c_p}$ and $\alpha_z=\frac{k_z}{\rho c_p}$ where α_x, α_y and α_z is the thermal diffusivity in the x,y,z-directions, respectively, and using the Crank-Nicolson method to carry out the time discretization, we obtain:

$$\left(\frac{T_{ijk}^{n+1}-T_{ijk}^{n}}{\Delta t}\right)=\frac{1}{2}\left(\alpha_x\frac{\partial^2 T}{\partial x^2}+\alpha_y\frac{\partial^2 T}{\partial y^2}+\alpha_z\frac{\partial^2 T}{\partial z^2}-u\frac{\partial T}{\partial x}-v\frac{\partial T}{\partial y}-w\frac{\partial T}{\partial z}\right)_{ijk}^{n+1}+\frac{1}{2}\left(\alpha_x\frac{\partial^2 T}{\partial x^2}+\alpha_y\frac{\partial^2 T}{\partial y^2}+\alpha_z\frac{\partial^2 T}{\partial z^2}-u\frac{\partial T}{\partial x}-v\frac{\partial T}{\partial y}-w\frac{\partial T}{\partial z}\right)_{ijk}^{n}$$

$$\Rightarrow -\frac{1}{2}\left(\alpha_x\frac{\partial^2 T_{ijk}^{n+1}}{\partial x^2}+\alpha_y\frac{\partial^2 T_{ijk}^{n+1}}{\partial y^2}+\alpha_z\frac{\partial^2 T_{ij}^{n+1}}{\partial z^2}+u\frac{\partial T_{ijk}^{n+1}}{\partial x}+v\frac{\partial T_{ijk}^{n+1}}{\partial y}+w\frac{\partial T_{ijk}^{n+1}}{\partial z}\right)+\frac{T_{ijk}^{n+1}}{\Delta t}=G \tag{2}$$

where

$$G=\frac{1}{2}\left(\alpha_x\frac{\partial^2 T_{ijk}^{n}}{\partial x^2}+\alpha_y\frac{\partial^2 T_{ijk}^{n}}{\partial y^2}+\alpha_z\frac{\partial^2 T_{ij}^{n}}{\partial z^2}-u\frac{\partial T_{ijk}^{n}}{\partial x}-v\frac{\partial T_{ijk}^{n}}{\partial y}-w\frac{\partial T_{ijk}^{n}}{\partial z}\right)+\frac{T_{ijk}^{n}}{\Delta t}.$$

In the applications 1 and 2 below, the initial value of the exact solution is taken as the initial condition, and boundary conditions are also specified by the above equation and change with time. However, for the applications 3 and 4, the conditions will be specified in the same case.

The following criteria will be used for the spatial discretization of the previous equation:

- For nodes with Δx and Δy distant from the boundary using the Central Difference Method with $O(\Delta x^2)$, we have:

$$\left(-\frac{\alpha_z}{2\Delta z^2}-\frac{w}{4\Delta z}\right)T_{i,j,k-1}^{n+1}+\left(-\frac{\alpha_y}{2\Delta y^2}-\frac{v}{4\Delta y}\right)T_{i,j-1,k}^{n+1}+\left(-\frac{\alpha_x}{2\Delta x^2}-\frac{u}{4\Delta x}\right)T_{i-1,j,k}^{n+1}+\left(\frac{\alpha_x}{\Delta x^2}+\frac{\alpha_y}{\Delta y^2}+\frac{\alpha_z}{\Delta z^2}+\frac{1}{\Delta t}\right)T_{ijk}^{n+1}+\left(-\frac{\alpha_x}{2\Delta x^2}+\frac{u}{4\Delta x}\right)T_{i+1,j,k}^{n+1}+\left(-\frac{\alpha_y}{2\Delta y^2}+\frac{v}{4\Delta y}\right)T_{i,j+1,k}^{n+1}+\left(-\frac{\alpha_z}{2\Delta z^2}+\frac{w}{4\Delta z}\right)T_{i,j,k+1}^{n+1}=G \tag{3}$$

where

$$G=\frac{\alpha_x}{2}\left(\frac{T_{i+1,j,k}^{n}-2T_{ijk}^{n}+T_{i-1,j,k}^{n}}{\Delta x^2}\right)+\frac{\alpha_y}{2}\left(\frac{T_{i,j+1,k}^{n}-2T_{ijk}^{n}+T_{i,j-1,k}^{n}}{\Delta y^2}\right)+\frac{\alpha_z}{2}\left(\frac{T_{i,j,k+1}^{n}-2T_{ijk}^{n}+T_{i,j,k-1}^{n}}{\Delta z^2}\right)-\frac{u}{2}\left(\frac{T_{i+1,j,k}^{n}-T_{i-1,j,k}^{n}}{2\Delta x}\right)-\frac{v}{2}\left(\frac{T_{i,j+1,k}^{n}-T_{i,j-1,k}^{n}}{2\Delta y}\right)-\frac{w}{2}\left(\frac{T_{i,j,k+1}^{n}-T_{i,j,k-1}^{n}}{2\Delta z}\right)+\frac{1}{\Delta t}T_{ijk}^{n}.$$

- Now, for the other nodes, using the Central Difference Method with $O(\Delta x^4)$, we have:

$$\begin{aligned}
&\left(\frac{\alpha_x}{24\Delta z^2}+\frac{w}{24\Delta z}\right)T_{i,j,k-2}^{n+1}+\left(-\frac{2\alpha_z}{3\Delta z^2}-\frac{2w}{3\Delta z}\right)T_{i,j,k-1}^{n+1}\\
&+\left(\frac{\alpha_y}{24\Delta y^2}+\frac{v}{24\Delta y}\right)T_{i,j-2,k}^{n+1}+\left(-\frac{2\alpha_y}{3\Delta y^2}-\frac{v}{3\Delta y}\right)T_{i,j-1,k}^{n+1}\\
&+\left(\frac{\alpha_x}{24\Delta x^2}+\frac{u}{24\Delta x}\right)T_{i-2,j,k}^{n+1}+\left(-\frac{2\alpha_x}{3\Delta x^2}-\frac{u}{3\Delta x}\right)T_{i-1,j,k}^{n+1}\\
&+\left(\frac{1{,}25\alpha_x}{\Delta x^2}+\frac{1{,}25\alpha_y}{\Delta y^2}+\frac{1{,}25\alpha_z}{\Delta z^2}+\frac{1}{\Delta t}\right)T_{ijk}^{n+1}\\
&+\left(-\frac{2\alpha_x}{3\Delta x^2}+\frac{u}{3\Delta x}\right)T_{i+1,j,k}^{n+1}+\left(\frac{\alpha_x}{24\Delta x^2}-\frac{u}{24\Delta x}\right)T_{i+2,j,k}^{n+1}\\
&+\left(-\frac{2\alpha_y}{3\Delta y^2}+\frac{v}{3\Delta y}\right)T_{i,j+1,k}^{n+1}+\left(\frac{\alpha_y}{24\Delta y^2}-\frac{v}{24\Delta y}\right)T_{i,j+2,k}^{n+1}\\
&+\left(-\frac{2\alpha_z}{3\Delta z^2}+\frac{w}{3\Delta z}\right)T_{i,j,k+1}^{n+1}+\left(\frac{\alpha_z}{24\Delta z^2}-\frac{w}{24\Delta z}\right)T_{i,j,k+2}^{n+1}=G
\end{aligned}\tag{4}$$

where

$$\begin{aligned}
G=&\frac{\alpha_x}{2}\left(\frac{-T_{i+2,j,k}^n+16T_{i+1,j,k}^n-30T_{ijk}^n+16T_{i-1,j,k}^n-T_{i-2,j,k}^n}{12\Delta x^2}\right)\\
&+\frac{\alpha_y}{2}\left(\frac{-T_{i,j+2,k}^n+16T_{i,j+1,k}^n-30T_{ijk}^n+16T_{i,j-1,k}^n-T_{i,j-2,k}^n}{12\Delta y^2}\right)\\
&+\frac{\alpha_z}{2}\left(\frac{-T_{i,j,k+2}^n+16T_{i,j,k+1}^n-30T_{ijk}^n+16T_{i,j,k-1}^n-T_{i,j,k-2}^n}{12\Delta z^2}\right)\\
&-\frac{u}{2}\left(\frac{-T_{i+2,j,k}^n+8T_{i+1,j,k}^n-8T_{i-1,j,k}^n+T_{i-2,j,k}^n}{12\Delta x}\right)\\
&-\frac{v}{2}\left(\frac{-T_{i,j+2,k}^n+8T_{i,j+1,k}^n-8T_{i,j-1,k}^n-T_{i,j-2,k}^n}{12\Delta y}\right)\\
&-\frac{w}{2}\left(\frac{-T_{i,j,k+2}^n+8T_{i,j,k+1}^n-8T_{i,j,k-1}^n-T_{i,j,k-2}^n}{12\Delta z}\right)+\frac{1}{\Delta t}T_{ijk}^n.
\end{aligned}$$

RESULTS AND DISCUSSION

The linear system generated by governing equation and the boundary conditions were solved via a Gauss-Seidel method, considering the maximum error for stopping criterion for the Gauss-Seidel of 10-8. All computations were run on a Intel Core i7/2.4G private computer using double precision arithmetic.

In the applications 1 and 2, the computational domain is $0 \le x, y, z \le 1$ with $t = 1$ and was used a uniform grid $\Delta x=\Delta y=\Delta z$ and was compared accuracy under the L_∞ norm errors, which is the maximum error in the entire domain, given by $\|e\|_\infty = \left|T_{(num)} - T_{(an)}\right|$, where $T_{(num)}$ and $T_{(an)}$ is the result of the numerical and analytical solution, respectively. Now, in the application 3, we used the L_2norm, defined by $\|\varepsilon\|_2 = \left[\left(\sum_{i=1}^{Nnost} e_i^2\right)/Nnost\right]^{1/2}$, where N_{nost} is the total number of nodes in the mesh and $e_i = \left|T_{(num)_i} - T_{(an)_i}\right|$.

Application 1

In this application,we considered a pure diffusive case according to the equation $\frac{\partial T}{\partial t}=\frac{1}{3}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$with an analytical solution given by $T(x,y,z,t)= e^{(t+x+y+z)}$. Table 1 shows the comparison between the present results and the analytical solution, which made an analysis of the L_∞ norm of error in the numerical solution of $T(x,y,z,t)$. The CPU time, according to the configurations mentioned above, is listed in Table 2.

Table 1. L_∞ norm of the error committed in T - Application 1.

Δt	**Δx = Δy = Δz**			
	1/10	**1/20**	**1/40**	**1/50**
0.1	2.18E-03	1.81E-03	1.79E-03	1.79E-03
0.05	9.29E-04	4.76E-04	4.50E-04	4.48E-04
0.01	6.25E-04	6.27E-05	1.91E-05	1.74E-05
0.005	6.16E-04	5.74E-05	6.13E-06	4.20E-06
0.001	6.13E-04	5.60E-05	4.40E-06	1.90E-06

In Table 1 is noted that the numerical results for some refined meshes was obtained a precision of at six decimal places, which is considered suitable for engineering.

Table 2. Computational time (s) - Application 1.

Δt	**Δx = Δy = Δz**			
	1/10	**1/20**	**1/40**	**1/50**
0.1	0.859	12.266	291.594	856.844
0.05	1.125	14.156	334.063	1019.172
0.01	2.219	19.203	439.656	1269.281
0.005	2.734	24.485	453.406	1494.672
0.001	8.844	52.343	709.250	2009.328

Application 2

Now, this is a advective-diffusive case according to the governing equation $\frac{\partial T}{\partial t}+2\frac{\partial T}{\partial x}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}$ which analytical solution is $T(x,y,z,t)=e^{(t+x+y+z)}$. Similarly to the previous application, the Table 3 and 4, show, respectively, the comparison between the present results and the analytical solution, which made an analysis of the L_∞ norm of error in the numerical solution of $T(x,y,z,t)$ and the CPU time.

Table 3. L_∞ norm of the error committed in T - Application 2.

Δt	**Δx = Δy = Δz**			
	1/10	**1/20**	**1/40**	**1/50**
0.1	4.92E-04	6.27E-04	6.35E-04	6.36E-04
0.05	1.36E-04	1.49E-04	1.57E-04	1.57E-04
0.01	2.11E-04	1.73E-05	4.96E-06	4.93E-06
0.005	2.13E-04	1.86E-05	1.30E-06	5.20E-07
0.001	2.14E-04	1.91E-05	1.49E-06	1.48E-06

Table 4. Computational time (*s*) - Application 2.

Δt	**Δx = Δy = Δz**			
	1/10	**1/20**	**1/40**	**1/50**
0.1	1.375	18.875	492.563	1547.500
0.05	1.906	27.766	778.922	2222.391
0.01	3.031	40.454	1136.250	3210.422
0.005	4.954	46.297	1273.188	3604.047
0.001	11.562	89.500	1535.500	4204.719

Application 3

In this application, was considered an advective-diffusive case with the governing equation

$$\frac{\partial T}{\partial t}+0{,}8\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}+\frac{\partial T}{\partial z}\right)=0{,}01\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

with the following analytical solution given by [Ge, Tian and Zhang (2013); Karaa (2006)]:

$$T(x,y,z,t)=(4t+1)^{-3/2}\exp\left(-\frac{1}{0{,}01(4t+1)}\right.$$
$$\left.\left((x-0{,}8t-0{,}5)^2+(y-0{,}8t-0{,}5)^2+(z-0{,}8t-0{,}5)^2\right)\right)$$

The Dirichlet boundary and initial conditions are directly taken from this analytical solution. The computational domain is $0 \le x,y,z \le 2$ and t = 1.25 and the regular mesh with $h = \Delta x = \Delta y = \Delta z$ was used.We adopt h = 0.025 so as to compare the results obtained in this work the ones presented by [Ge, Tian and Zhang (2013)] in which three methodologies were implemented for the solution of convection-diffusion equation, all based on the alternating direction implicit method. We may note from Table 5, the results of this study in comparison with the literature, have an equal or higher accuracy for the different values of λ. Even considering a coarse mesh (h = 0.05), the methodology proposed here achieves a precision equivalent to those obtained by other authors with a very refined mesh.

Table 5. L_2error with h = 0.025 and h = 0.05, $\Delta t = \lambda h^2$, t = 1.25and different λ - Application 3.

	***h* = 0.025**				***h* = 0.05**
λ	**Douglas–Gunn ADI scheme [Ge, Tian and Zhang (2013)]**	**Karaa's ADI scheme [Ge, Tian and Zhang (2013)]**	**Exponential high order compact ADI scheme [Ge, Tian and Zhang (2013)]**	**Present work**	**Present work**
5	5.599E-04	5.184E-05	5.266E-05	5.261E-06	4.900E-05
10	5.764 E-04	5.268E-05	5.309E-05	1.353E-05	5.431E-05
20	5.991E-04	6.978E-05	6.674E-05	4.845E-05	8.152E-05
40	7.329E-04	2.035E-04	1.858E-04	1.907E-04	2.413E-04

Table 6. Computational time (*s*) - Application 3.

	***h* = 0.025**				***h* = 0.05**
λ	**Douglas–Gunn ADI scheme [Ge, Tian and Zhang (2013)]**	**Karaa's ADI scheme [Ge, Tian and Zhang (2013)]**	**Exponential high order compact ADI scheme [Ge, Tian and Zhang (2013)]**	**Present work**	**Present work**
5	762.04	1081.65	829.45	897.953	121.344
10	379.87	551.39	412.14	476.562	72.203
20	188.42	280.35	207.96	316.64	41.875
40	97.45	142.28	104.11	138.891	28.36

Application 4

In this case, we considered the equation $\frac{\partial T}{\partial t}+w\frac{\partial T}{\partial z}-\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=0.$ Considering T_{air} = 300 K, we adopt the following thermophysical properties [Incropera and DeWitt (1996)]: ρ = 1.1614 kg/m^3, c_p= 1007 J/kg.K, k = 0.0263 W/m.K for mass density, heat capacity and thermal conductivity, respectively. Thus, $\frac{k}{\rho c_p}\cong 0.00022488$. A parabolic velocity profile, representing a fully developed laminar flow was defined by [Romão and Moura (2012)]: $w(x,y,z,t)=\frac{16U_{max}}{L_x\cdot L_y}\left(-\frac{x^2}{L_x}+x\right)\left(-\frac{y^2}{L_y}+y\right)$, where U_{max} is the top speed on the flow centerline.

We assume air at atmospheric pressure and temperature 300 K, flowing into a rectangular channel with dimensions $L_x \times L_y \times L_z$, considering the following boundary conditions, according the Fig. 1:

- in the plane XY with z = 0: $T = T_{inlet}$, being T_{inlet} the inlet temperature in the channel;
- in the plane XY with $z = L_z$: $\frac{\partial T}{\partial z}=0$;
- in the plane YZ with x = 0 or $x = L_x$: $\frac{\partial T}{\partial x}=0$;
- in the plane XZ with y = 0: $\frac{\partial T}{\partial y}=0$;
- in the plane XZ with $y = L_y$: $T = T_c$, being T_c the cooling temperature of fluid.

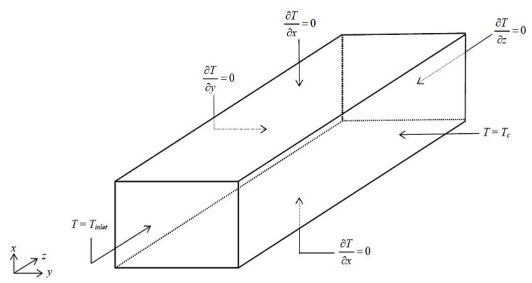

Figure 1. Computational domain and boundary conditions - Application 4.

Considering an inlet temperature of 30º C, our objective is the cooling of air at an average temperature of 15º C in the outlet section of the channel. Taking $L_x = L_y$ = 0.2 m; L_z = 1 m and L_t = 100 s and considering a mesh refinement $\Delta x = L_x/40$, $\Delta y = L_y/40$, $\Delta z = L_z/100$ and $\Delta t = L_t/500$. By varying the values of the cooling temperature (T_c) at the entrance, the results of average temperature in the output section of the channel, here denoted by T_a, are shown in Table 7, considering the order of accuracy with two decimal places.

In Table 7, we note that as the cooling temperature decreases by 1°C, the average temperature of the output section is reduced to 0.16 °C. Thus, in order to achieve a mean temperature of 15 °C in the output section will require a cooling temperature of approximately -64.25 ºC.

Table 7. Results of T_a at the outlet section of the channel depending on thecooling temperature in the entrance (T_c), considering $\Delta x = L_x/200$,$\Delta y = L_y/10$, $\Delta z = L_z/100$ and $\Delta t = t/500$.

T_c(ºC)	T_a (ºC)
10	26.78
9	26.62
8	26.46
7	26.30
6	26.14
5	25.98
4	25.82
3	25.66
2	25.50
1	25.34
0	25.18
-1	25.02
-2	24.96
-3	24.80

Two other strategies for the expected average temperature is achieved on the output variation were the length and size of the inlet section of the channel. Thus, taking $L_x = L_y$ = 0.2 m and varying L_z and L_t = 100 s,considering a mesh refinement $\Delta x = L_x/40$, $\Delta y = L_y/40$, Δz = 0.01 and $\Delta t = L_t/500$, with T_c = 1º C, the obtained numerical values shown in Table 8. Note, that it was necessary length z = 3 m so that the temperature of 23.35 ° C was reached. Likewise, to reach 22.22°C, it was necessary to z = 6 m, indicating the relation between the variation of the channel length and cooling the same. It can be noted clearly, that for the conditions in this case, the variation of the channel length, the average temperature tends to stagnate at a value close to 22 °C.

Table 8. Results of T_a at the outlet section of the channel depending on the variation of the channel length, considering $\Delta x = L_x/40$, $\Delta y = L_y/40$, Δz = 0.01 and $\Delta t = L_t/500$.

L_z (m)	T_a (ºC)
1.0	25.34
1.5	24.67
2.0	24.14
2.5	23.71
3.0	23.35
3.5	23.06
4.0	22.81

4.5	22.61
5.0	22.45
5.5	22.32
6.0	22.22
6.5	22.16

Considering, now, the variation of the inlet section of the channel and taking $L_x = L_y$; varying L_z= 1 m and L_t = 100 sand considering a mesh refinement $\Delta x = \Delta y = 0.005$, $\Delta z = L_z/100$ and $\Delta t = L_t/500$, with T_c = 1º C, we obtained the values shown in Table 9.It can be noted that for L_z = 1 m, $L_x = L_y$ = 0.1 m, T_c = 1ºC and L_t = 100 s was achieved $T_a \cong$ 22.45ºC. However, modifying L_t = 200 s or L_t = 300 s, the results in the output section have not changed, which proves the steady state temperature distribution.
Another test was carried out by taking L_z = 5m, $\Delta z = L_z/500$, $L_x = L_y$ = 0.1 m, $\Delta x = \Delta y = L_x/20$, L_t = 100 s, $\Delta t = L_t/500$ to T_c = 1ºC. The results obtained in this case are shown in Fig. 2. It can be noted that the mean temperature of 15° C was achieved in 100s.

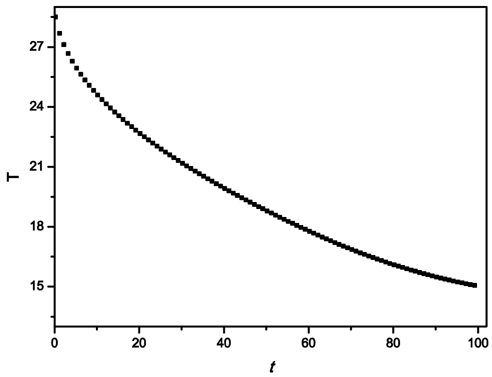

Figure 2. Results of the average temperature in the outlet section of the channel depending of time - Application 4.

Table 9. Results of T_a at the outlet section of the channeldepending on the variation of the channel length,considering $\Delta x = L_x/40$, $\Delta y = L_y/40$, $\Delta z = 0.01$ and $\Delta t = L_t/500$.

$\Delta x = \Delta y$	T_a (ºC)
0.20	25.98
0.19	25.16
0.18	24.98
0.17	24.77
0.16	24.54
0.15	24.28
0.14	23.98
0.13	23.65
0.12	23.26
0.11	22.80
0.10	22.25

CONCLUSIONS

In this paper, we propose a high-order Finite Difference Method for three-dimensional heat transfer equation. Numerical examples show that the method can be used to simulate the numerical solution of the equation, with emphasis on the application involving heat exchange in a rectangular channel. By observing the detailed comparison of numerical and analytical results, it is convinced that the proposed scheme is very simple, stable and accurate for the solutions of the heat transfer equation.

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