A BRIEF STUDY ABOUT TWO-PHASE FLOW (LIQUID + GAS) MATHEMATICAL MODELING

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Received: August 30, 2012
Revised: September 30, 2012
Accepted: October 30, 2012

ABSTRACT

This work presents a brief study about mathematical modeling of two-phase flow (liquid + gas), approaching homogeneous phase flows and heterogeneous phase flows, in addition to mathematical modeling of pressure drop in flow restriction through abrupt expansions and abrupt contractions. Also presents a summary of flow patterns main types and a brief study about how the flow velocity influences these patterns.

Keywords: two-phase flow, mathematical modeling, hydrodynamics, flow patterns, pressure drop.

NOMENCLATURE

A_i	cross-sectional flow area, m²
A_{VC}	vena contracta área ratio, %
c	flow factor
C_0	concentration parameter
C_c	contraction coefficient
D_e	equivalent diameter of flow channel, m
E_i	energy, J
F	body force, kgf
G	mass flux, kg/s
g	gravity acceleration, m/s²
g_c	conservation ratio, %
J	mixture average superficial velocity, m/s
n_G	normal vector in gas phase direction
P	pressure, Pa
P_c	contraction flow region pressure, Pa
P_e	expansion flow region pressure, Pa
q_i	heat flux, J/s
S	slip ratio
$\bar{\bar{U}}$	metric tensor of the space
u_i	flow velocity, m/s
u_{GJ}	sliding velocity, m/s
u_G	gas phase drift velocity, m/s
u_L	liquid phase sliding velocity, m/s
u_{GM}	diffusion velocity (gas velocity relative to the velocity of center of mass), m/s
u_{GS}	full velocity of two-phase flow, m/s
v_{fg}	specific volume change during evaporation, m³/kg
J=u_G ?_G+u_L ?_L	liquid phase specific volume change, m³/kg
$\bar{v}$	average specific volume of the mixture at the location considered, m³
W_G	gas phase total flow rate, kgf/s
W_L	liquid phase total flow rate, kgf/s
X	parameter group
Z	flow parameter

Greek symbols

α_G	gas phase void fraction
α_L	liquid phase void fraction
ρ_M	mixture specific mass, kg/m³
ρ_G	gas phase specific mass, kg/m³
ρ_L	liquid phase specific mass, kg/m³
$\bar{\bar{\tau}}_i$	stress tensor
$\varnothing_i$	mass flux across the interface
σ	surface tension, kgf/cm²
μ_G	gas phase viscosity, kg/(m.s)

μ_L	liquid phase viscosity, kg/(m.s)

Subscripts

M	refers to mixture (liquid phase + gas phase)
G	refers to gas phase
L	refers to liquid phase

INTRODUCTION

The study of multiphase flow has become more and more important in a wide variety of engineering systems due to incessant need to optimize performance and improve the security of these systems. Some of these studies main applications are listed below: exchange heat systems (heat exchangers, cooling towers); transportation systems (ejectors, pipelines of oil-gas mixtures, compressors and pumps); power generation systems (pressurized water nuclear reactors, liquid metal nuclear reactors, geothermal energy facilities, internal combustion engines, jet engines).

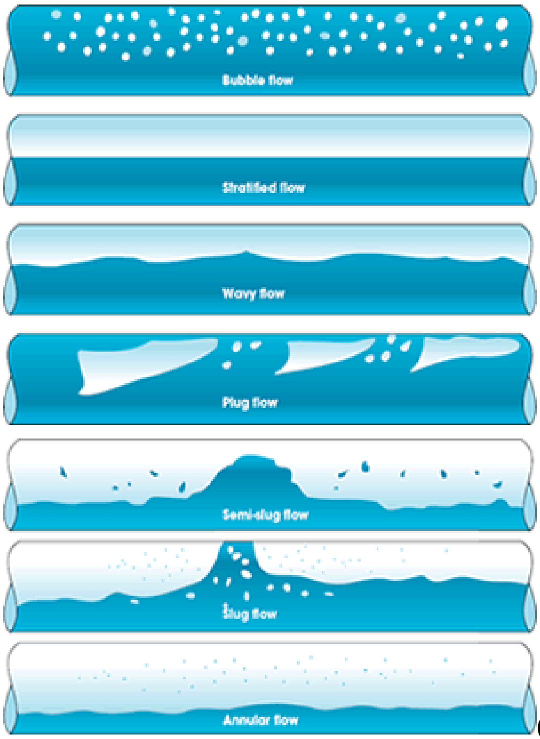
The final purpose of the two-phase flow study is to determine the characteristics of heat transfer and pressure drop for a given flow. One of the main interface conditions is the presence or absence of the heat transfer, since the occurrence of heat transfer causes a phase change and consequently a change in distribution of the flow pattern, which moreover causes a hydrodynamic change, since a pressure drop along the flow field affects the heat transfer characteristics. It is worth to observe a two-phase flow within a pipe hardly will become fully developed at low pressure because of changes of the large bubbles shape, inherent to pressure drop along the pipe, which continuously changes the state of the fluid and thereby change the phase distribution and the flow pattern.

These observations suggest a highly complex in two-phase flow that suffer heat transfer where a local description, or point description, is insufficient without knowledge of the flow "history". Hydrodynamics instabilities and occasional departures from the balance area between phases add complexity factors to flow. With the purpose to avoid these complications many analyzes are based on the assumption of fully developed flow patterns and without heat addition to flow.

The two-phase flow hydrodynamic behavior – such as pressure variation, velocity distribution and void factions – varies systematically with observed flow regime, as well as a single-phase flow, whose behavior depends only of the flow regime (laminar or turbulent). However, in contrast to single-phase flow, there is a lack of liquid-gas two-phase flow generalization principles that serve as the basis for practical problems solving. Therefore, the identification of the flow regime automatically provides the location of the phase boundaries which allows the calculation of the integral forms of the continuity equation and momentum equation.

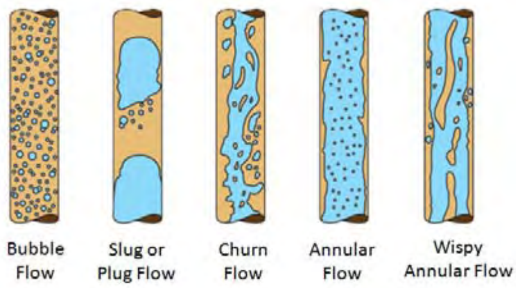
ADIABATICS TWO-PHASE FLOW MAIN PATTERNS

Figures 1a and 1b show two-phase flow patterns (liquid + gas) in horizontal and vertical pipes, respectively.



(a)

Figure 1a. Two-phase flow patterns in horizontal pipes. (Available at: <*http://beta.globalspec.com/ reference /9778/349867/dealing-with-two-phase-flows*>, Accessed on: *May 28, 2012*).



(b)

Figure 1b. Two-phase flow patterns in vertical pipes. (Available at: <*http://dpwsd.waterworld.com/index/ display/article-display/3887904678/articles/membranes/volume-2/issue-30/Features/Importance_of_Air_ Scour_in_Membrane_Bioreactor_Systems. html*>, Accessed on: *May 28, 2012*).

In the *bubble flow* occurs the dispersion of gas bubbles within the liquid phase, featuring de flow continuous way – the bubbles stay concentrated at the top of the pipe. In the *plug flow* there is a displacement of large bubbles – called *plugs* or *pockets* – at the top of the pipe, and may or may not be small bubbles in the region immediately below these *plugs*. The existence of two distinct phases – the gas phase at the top of the pipe and the liquid phase at the bottom of the pipe – separated by a relatively smooth interface features the *stratified flow*. If the two phases are separated by an irregular and wavy interface – always associated with high velocities of the gas phase – the flow receives the *wavy flow* classification. If the gas phase velocity is high enough to causes a rupture in the interface, at some point the liquid phase will segregate the gas phase until it reaches the top of the pipe, creating large pockets of gas – also called *slugs*. This phenomenon characterizes the flows classified as *slug flows*. The *annular flow* gets its name from the annular film formed by liquid phase in the pipeline wall which surrounds the gas phase inside. The liquid film thickness is not constant and the interface that segregates the two phases is unstable, allowing a portion of the liquid phase remains suspended – in a filaments form – within the gas phase. In the *spray flow* , the gas phase is the flow continuous way which drags the liquid phase in its inner in the form of droplets.

Symmetric patterns are more likely to occur in vertical flown than in horizontal flows. There are no *stratified flow* in vertical flow because the gravity action and the flow relative direction.

The *stratified flow* has a stronger tendency to occur in horizontal flow with ratio speed of two phases relatively low. If flow velocity is increased, waves begin to form on the surface of the liquid phase, and these waves can grow until it reaches the top of the pipe. When this scenario occurs, the gas phase is awhile throttled, or even locked, which discontinues the flow, leading to the formation of elongated bubbles, also called “*slugs*”. These elongated bubbles should be avoid because they produce significant pressure fluctuations, which may cause an unequally distribution of liquid phase and gas phase among industrial facilities causing tanks flooding. The increase of the liquid phase velocity, in the intermittent form, may accelerate the corrosion effects.

INFLUENCE RELATION BETWEEN FLOW SPEED AND FLOW PATTERN

The main difference between horizontal and vertical two-phase flows is in the vertical flows there are no “underside of the pipe” where the denser fluid prefer to stay, which precludes the existence of the *stratified flow* in vertical pipelines. As the pipelines usually follow the ground slope, the complexity of determining the flow regime is often greater than what shown in this work.

For this reason, flow regime maps as shown in Fig. 2 are very useful to obtain information about the mechanisms that create these flow regimes. The horizontal axis displays the gas phase superficial velocity values while the vertical axis displays the liquid phase superficial velocity values.

As the velocities close to zero, the pipeline acts as a long horizontal tank with the liquid phase at the bottom and the gas phase at the top. Increasing the gas phase velocity, waves begin to form on the liquid phase surface. Due to friction between the two phases, the increase of the gas phase velocity will drag the liquid phase in the flow direction thereby reducing the liquid phase level. Further to increase the gas phase velocity he generated turbulence is intensified until a liquid phase surface rupture occurs causing liquid phase droplets being entrained through the gas stream while the horizontal surface existed before bends inside the pipe until to cover the entire inner surface as a liquid film. The droplets are entrained by the gas phase turbulence until they occasionally collide with the pipe wall and then they are deposited in the liquid film that covers the internal walls.

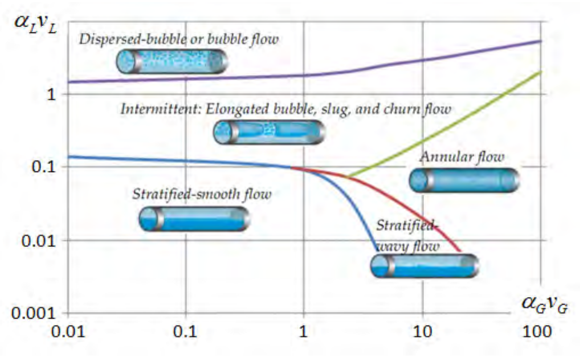


Figure 2. Generic two-phase flow regime map. (Available at: <*http://drbratland.com/PipeFlow2/ chapter1.html*>, Accessed on: *May 30, 2012*).

If the liquid flow is too high, the turbulence will be strong and any gas particle will tend to interfere in the liquid phase in the form of small bubbles. For slower liquid flows, the bubbles will float toward the top of the pipe, forming an agglomerate. A suitable mixture of liquid and gas may form de *Taylor-bubble* – name given to the large bubbles that separate the liquid *slugs*.

If the gas flow is maintained sufficiently high, it will drag the fluid so quickly that liquid phase will be maintained low throughout the entire pipeline, preventing the formation of *slugs*. It is possible to take advantage of that and create operational parameters that define how oil-gas pipeline must be operated, usually setting the minimum gas rate needed to ensure a *slug* free flow. It is important to note that flow regime maps are useful tools for generating regime flow overviews that can be generated from private input data, however, each map is not general enough to validate other flows input data.

TWO-PHASE FLOW HOMOGENEOUS PHASE MATHEMATICAL MODELING

Used for mixed flow approach, such as *bubble flow* and *slug flow*, this model treats the mixture as a whole, assuming the liquid phase and the gas phase have the same flow velocity. There are six parameters to be determined (thee-dimensional velocity vector, pressure, temperature and specific mass) and as in the single-phase flow the number of unknowns and the amount of available equations is equal (three momentum conservation equations, mass conservation equation, energy conservation equation and equation of state).

In fact, the sliding velocity between the two phases cannot be ignored and slip model must be introduced as a homogeneous model improvement, according Tong & Tang (1997). In this model, the continuity equations (mass conservation), momentum conservation equations and energy conservation equation are written to whole mixture. Then it is written one another continuity equation to only one phase, usually the gas phase. To allow the sliding velocity become located between the two phases, the sliding velocity u_{GJ}, or diffusion velocity u_{GM} (gas velocity relative to the velocity of center of mass), is defined as:

$$U_{GJ} = u_G - J \tag{1}$$

Once the mixture surface average velocity is constant to single-phase flow, so

$$J = u_G \alpha_G + u_L \alpha_L \tag{2}$$

and

$$u_{GM} = u_G - \frac{G}{\rho_M} \tag{3}$$

when the mixture velocity will be $\frac{G}{\rho_M}$, the mixture mass flow will be given by

$$G = u_G \alpha_G \rho_G + u_L \alpha_L \rho_L \tag{4}$$

and the mixture specific mass will be given by:

$$\rho_M = \alpha_G \rho_G + \alpha_L \rho_L \tag{5}$$

According Hsu & Graham (1976), using these definitions and integrating the entire pipeline cross section, with some manipulations, the Zuber’s kinematic equation results in

$$\frac{\langle u_{GS} \rangle}{\langle \alpha_G \rangle} = C_0 \langle J \rangle + \frac{\langle u_{GJ} \alpha_G \rangle}{\langle \alpha_G \rangle} \tag{6}$$

where C_0 is given by:

$$C_0 = \frac{\langle \alpha_G J \rangle}{\langle \alpha_G \rangle \langle J \rangle} \tag{7}$$

This model is considered a widespread homogeneous model and serves quite well to describe complex systems, such as nuclear reactors for example.

TWO-PHASE FLOW HETEROGENEOUS PHASE MATHEMATICAL MODELING

Used for flow with separated phases, such as *stratified flow* and *annular flow*, in which the two phases occupy the whole flow field simultaneously. Two sets of conservation equations corresponding to two phases must be complemented by a set of discontinuity conditions. According Ishii & Hibiki (2011), a topological additional law relative to the void fraction, for phase unknowns, is necessary to compensate the information loss during the model simplification.

The intrinsic constitutive laws (equations of state) are those of each phase. The external constitutive laws are four transfer laws on the walls (friction and mass transfer to each phase) and three interface transfer laws (mass, momentum and energy).

The equations proposed to two-phase flows by Vernier & Delhaye (1968) are shown below.

Mass conservation:

$$\frac{\partial \rho_i}{\partial t} + div\left(\rho_i u_i\right) = 0 \tag{8}$$

Momentum consevation:

$$\frac{\partial \rho_i u_i}{\partial t} + div\left(\overline{\overline{\tau}}_i\right) + \rho_i F \tag{9}$$

Energy consevation:

$$\frac{\partial}{\partial t}\left[\rho_i\left(\frac{u_i^2}{2} + E_i\right)\right] + div\left(\rho i\right)\left(\frac{u_i^2}{2} + E_i\right)u_i = div\left(\overline{\overline{\tau}}_i u_i - q_i\right) + \rho_i F u_i \tag{10}$$

These equations are subjected to the following boundary conditions.

Mass conservation:

$$\sum_{i=L,G} \phi_i = 0 \tag{11}$$

Momentum consevation:

$$\sum_{i=G,L} \phi_i - div\left(\sigma \bar{\bar{U}}\right) - \sigma n_G \left(\frac{2}{R}\right) = 0 \tag{12}$$

Energy consevation:

$$\sum_{i=G,L} \phi_i - div\left(u_i \bar{\bar{U}}\right) - \sigma u_i \left(\frac{2}{R}\right) n_G = 0 \tag{13}$$

LOCAL PRESSURE DROP IN TWO-PHSE FLOWS

As the two-phase flow pressure variation is directly related to the flow pattern the majority of studies are focused on the local analyzes of the pressure variation in the flows with well defined patterns. The desired pressure drop forecast is used generally throughout the whole flow channel and includes several different flow patterns when there is a heat transfer condition. Thus, assuming the phases are in the thermodynamic equilibrium, it is necessary to use the sum of the local pressures ΔP. The temperature increase causes phase changes throughout the flow channel, increasing the gas region and varying the momentum and the flow velocity.

The pressure drop in two-phase flow usually boils down to three components: frictional loss, momentum change and elevation pressure drop arising from the effect of the gravitational force field. The sum of the local pressure ΔP is normally written as:

$$\left(\frac{\partial P}{\partial z}\right)_{system} = \left(\frac{\partial P}{\partial z}\right)_{fi} + \left(\frac{\partial P}{\partial z}\right)_{mc} + \left(\frac{\partial P}{\partial z}\right)_{epd} \tag{14}$$

The pressure gradient of the momentum change is given by

$$\left(\frac{\partial P}{\partial z}\right)_{mc} = \frac{G^2}{g_c}\frac{d\bar{v}}{dz} \tag{15}$$

being the average specific volume of the mixture at the location considered given by:

$$\bar{v} = \frac{W_G v_G + W_l v_L}{W_G + W_L} = v_L \left[1 + \left(\frac{X}{v_L}\right)\left(v_G - v_L\right)\right] \tag{16}$$

and the elevation pressure gradient is given by:

$$\left(\frac{\partial P}{\partial z}\right)_{edp} = \frac{1}{\bar{v}}\left(\frac{g}{g_c}\right) \tag{17}$$

The pressure drop for a given channel length becomes.

$$\text{P}=\int_0^{\text{L}} \left(\frac{\partial P}{\partial z}\right)_{system} dz \tag{18}$$

PRESSURE DROP IN TWO-PHASE FLOW RESTRICTION THROUGH ABRUPT EXPANSION

Assuming a single-phase incompressible fluid one-dimensional flow, a conservation energy equation will be

$$\text{P}_1 + \frac{\rho u_1^2}{2g_c} = P_2 + \frac{\rho u_2^2}{2g_c} + K\left(\frac{\rho u_1^2}{2g_c}\right) \tag{19}$$

where K may be obtained as a momentum balance as

$$K = \left(1 - \frac{A_1}{A_2}\right)^2 \tag{20}$$

and the subscripts 1 e 2 refer to the positions before and after the restriction, respectively.

Combining the continuity equation and equation (19) yields the total pressure change

$$P_2 - P_1 = \frac{+2A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right)\left(\frac{\rho u_1^2}{2g_c}\right) \tag{21}$$

where (+) signal indicates pressure recovery after the expansion.

For frictionless flow, the pressure rise is due the momentum or velocity change only, which may be obtained from:

$$\left(\Delta P\right)_{mc} = \left[1 - \left(\frac{A_1}{A_2}\right)^2\right]\left(\frac{\rho u_1^2}{2g_c}\right) \tag{22}$$

Hence the expansion loss variation is

$$\begin{aligned}\left(\Delta P\right)_{fl} &= \left(\Delta P\right)_{mc} - \left(P_2 - P_1\right) \\ &= \left[1 - 2\left(\frac{A_1}{A_2}\right) + \left(\frac{A_1}{A_2}\right)^2\right]\left(\frac{\rho u_1^2}{2g_c}\right)\end{aligned} \tag{23}$$

For two-phase flow, additional assumptions are made that thermodynamic phase equilibrium exists before and after the restriction, and that no phase change occurs over the same point of restriction. Thus, the momentum variation equation across an abrupt expansion is

$$P_1 + A_1 + \frac{W_{L1}u_{L1}}{g_c} + \frac{W_{G1}u_{G1}}{g_c} = P_2 A_2 + \frac{W_{L2}u_{L2}}{g_c} + \frac{W_{G2}u_{G2}}{g_c} \tag{24}$$

and liquid and gas flow continuity equations, assuming the full mass flow rate to be liquid, are:

$$\mathrm{w}_{L2} = \mathrm{w}_{L1} = w(1-X) = \bar{\rho} u A_1 (1-X) \tag{25}$$

$$\mathrm{w}_{G2} = \mathrm{w}_{G1} = wX = \bar{\rho} u A_1 X \tag{26}$$

Additional continuity considerations and the definition of void fraction give

$$u_{L1} = \frac{u(1-X)}{1-\alpha_1} \tag{27}$$

$$u_{L2} = \frac{(A_1/A_2)\,u(1-X)}{1-\alpha_2} \tag{28}$$

$$u_{G1} = \frac{uX\rho_L}{\alpha_1 \rho_G} \tag{29}$$

$$u_{G2} = \frac{(A_1/A_2)(uX)\,\rho_L}{\alpha_2 \rho_G} \tag{30}$$

Combining these equations, the static pressure change will be given by:

$$P_2 - P_1 = \left(\frac{A_1}{A_2}\right)\left(\frac{\rho_L u^2}{g_c}\right)\left\{X^2\left(\frac{\rho_L}{\rho_G}\right)\left(\frac{1}{\alpha_1} - \frac{A_1}{A_2\alpha_2}\right) + (1-X)^2\left[\left(\frac{1}{1-\alpha_1}\right) - \frac{A_1}{A_2(1-\alpha_2)}\right]\right\} \tag{31}$$

Assuming the void fractions upstream and downstream from expansion are the same, this equation may be rewrite as

$$\Delta P_e = P_2 - P_1 = \left(\frac{A_1}{A_2}\right)\left(\frac{\rho_L u^2}{g_c}\right)\left(1 - \frac{A_1}{A_2}\right)\left[\frac{X^2\rho_L}{\alpha\rho_G} + \frac{(1-X^2)}{1-\alpha}\right] \tag{32}$$

and the relation between X and α express by

$$\frac{1}{X} = 1 - \frac{\rho_L}{\rho_G\,(1-c/\alpha)} \tag{33}$$

where c is defined by

$$c = \alpha + \frac{(1-\alpha)}{S} \tag{34}$$

According Tong & Tang (1997, p. 212), the c flow factor may be written as a Z parameter function, so that

$$Z = \left[\frac{D_e G}{\mu_L(1-\alpha) + \mu_G \alpha}\right]^{1/6}\left[\frac{G^2}{(\rho_{mi})^2 g D_e}\right]^{1/8}\left[\frac{(1-X)\rho_G + X\rho_L}{(1-X)\rho_G}\right]^{1/4} \tag{35}$$

assuming no sliding between the two phases.

PRESSURE DROP IN TWO-PHASE FLOW RESTRICTION THROUGH ABRUPT CONTRACTION

The two-phase flow pressure drop at an abrupt contraction usually may be predicted by using a homogeneous flow model and the *Kays-London* single-phase pressure coefficient. The mixture is considered homogenized due to mixing effect exists along the jet caused by the point of contraction.

The follow homogeneous flow model describes the pressure drop, in 200-600 PSI two-phase water-air flow, caused by abrupt contraction points:

$$\Delta P_c = \left(1 + \frac{Xv_{fg}}{v_L}\right)^{-1} \rho_L u^2 \left[\frac{1-(A_1/A_2)^2 + K_c}{2g_c}\right] \tag{36}$$

Where

$$K_c = \left[1/C_c - 1\right]^2 \tag{37}$$

The total pressure drop across a contraction point may be approximated by the equation:

$$\Delta P_c = \left(\frac{A_2^2 G^2}{2 g_c A_1^2}\right)\left[\left(\frac{1}{A_{VC}} - 1\right)^2 + \left(1 - \frac{A_1}{A_2}\right)^2\right] \left[\frac{X}{\rho_G} - \frac{(1-X)}{\rho_L}\right] \quad (38)$$

CONCLUSIONS

In this paper was made a brief description of importance of two-phase flow (liquid + gás) mathematical modeling study.

The main two-phase flow patterns in the horizontal and vertical pipelines are shown in figures 1A and 1B.

Figure 2 illustrates a two-phase flow regime map, a very useful tool to obtain information about mechanisms that create the flow regimes over a given pipeline – once the flow patterns are mostly influenced by the pipeline position and the flow velocity.

Also became a brief description of how the flow pattern is modified by increase or reduce of the two-phase flow phases (liquid phase and gas phase) velocities.

In sequential were presented the mathematical models that govern the two-phase flow local pressure drops, and also the mathematical models that govern the two-phase flow pressure drops through abrupt expansion and contractions.

REFERENCES

Hsu, Y. Y., and Graham, R. W., 1976, *Transport Processes in Boiling and Two-Phase Systems*, 1st Edition, Hemisphere.

Ishii, M., and Hibiki T., 2011, *Thermo-Fluid Dynamics of Two-Phase Flow*, 2nd Edition, Springer.

Tong, L. S., and Tang, Y. S., 1997, *Boiling Heat Transfer and Two-Phase Flow*, 2nd Edition, Taylor & Francis.

Vernier, P., and Delhaye, J. M., 1968, *General Two-Phase Flow Equations Applied to the Thermohydrodynamics of BWR's*, 1st Edition, Centre d'Estudes Nucleaires de Grenoble, Service des Transports Thermiques.

Zuber, N., 1958, *On Stability of Boiling Heat Transfer*, 1st Edition, ASME.