THE ASSIMPTOTICAL SOLUTION FOR THE STUDY OF THE TRANSITION OF A GAS-LIQUID FLOW FROM COUNTER-CURRENT TO CO-CURRENT

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ABSTRACT

The transition of a flow of a liquid film from counter-current to co-current to a gas flow is known as the flooding phenomenon. In this paper is shown a quite criterions mathematical formulation in the form to identify with success the flooding point. One applies thus the conservation equations for a bi-dimensional and isothermical flow. Using the theorem of the PI of Vashy-Buckingham one makes a dimensional analysis in order to obtain parameters that make it possible to establish an asymptotic analysis for the a-dimensional equations and reduce a set of PDEs to a unique PDE for the thickness of the liquid film. This PDE will be decomposed in an equation for the permanent problem and another referring to the transient. It will be developed only the non–linear permanent equation applying the spectral method of collocation of Chebychev for its discretization. One intends in this paper to stress the physical interpretation of the equations for the thickness of the liquid film obtained through the asymptotic expansion and describe the characteristics of the used numeric method (spectral technique). The results are compared to others found in the literature proving that the spectral method of collocation is a very powerful technique for the solving of this kind of problem.

Keywords: Flooding Phenomena, Spectral collocation Chebyshev Method, Asymptotic expansion.

INTRODUCTION

The flooding phenomenon that occurs in a vertical flow of a liquid film, subject to a counter-current gas flow has been studied 40 years ago (Chang, 1986). The understanding of this phenomenon is very important to solver different problems that occur in Chemical, Petroleum and Nuclear Engineering (Ataídes, 1994 and Biage, 1989).

The flow of a liquid film on the canal wall, in the presence of a gas, flowing counter-current can exist only for an interval of gas flows below a certain limit for a fixed liquid flow. Above this limit it is experienced a strong decrease of the liquid flow descending in the form of a film. The set composed by the critical gas flow plus the liquid flow that corresponds to this limitation is considered the definition of what is called the flooding point (Ataídes, 1994 and Biage, 1989).

The term flooding has been used to describe various aspects of the transition of the liquid film - initially flowing counter-current – to a co-current flow to the gas flow. But the definition of flooding considering an injection as being done through a porous wall is given as being the point (pair injected liquid flow – gas flow) before which the total of the liquid flows in the direction of the base of the experimental section and after which part of the liquid is taken in the direction of the upper part of this section forming a flow of an co-current flow and continuing the other part flowing in the direction of the base counter –current to the gas flow (Ataídes, 1994).

This definition given to flooding corresponds in reality to a transition in the structure of the descending liquid film. This change constitutes an easier parameter to preview in a theoretical model- making.

MATHEMATICAL FORMULATION

The Figure 1 shows schematically a liquid film flowing on a wall of a rectangular tube, counter-current to the flow of a gas.

To proceed the mathematical formulation of this system are used the following assumptions: bi-dimensional flow, Newtonian fluid, isotherm and without change of phase with constant superficial tension and absence of sliding on the wall.

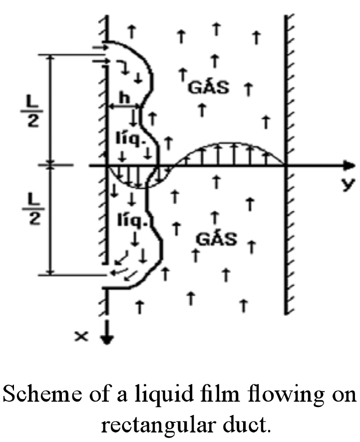

Figure 1. Scheme of a liquid film flowing on a wall of a rectangular duct.

The system of the conservation equations used in this study was dimensioned through the groups obtained by the theorem of the PI of Vaschy-Buckingham (Ataídes, 1994 and Biage, 1989). These groups are:

$$u^* = \left(\frac{gQ^2}{\nu}\right)^{1/3} u; \quad v^* = \varepsilon\left(\frac{gQ^2}{\nu}\right)^{1/3} v \tag{1}$$

$$x^* = \frac{1}{\varepsilon}\left(\frac{\nu Q}{g}\right)^{1/3} x; \quad y^* = \left(\frac{\nu Q}{g}\right)^{1/3} y;$$

$$t^* = \frac{1}{\varepsilon}\left(\frac{\nu^2}{g^2 Q}\right)^{1/3} t \tag{2}$$

$$h^* = \left(\frac{\nu Q}{g}\right)^{1/3} h; \quad P^* = \frac{1}{\varepsilon}(\rho^2 g^2 \mu Q)^{1/3} P \tag{3}$$

$$\sigma^* = \frac{1}{\varepsilon^3}(\rho g \mu^2 Q^2)^{2/3}\sigma; \quad \tau^{*yx} = \mu\left(\frac{g^2 Q}{\nu^2}\right)^{1/3}\tau^{yx} \tag{4}$$

The variables u and v are the velocities in the directions x and y, respectively, g is the gravity acceleration, ρ is the density of the liquid, ν is the cinematic viscosity, P is the pressure, τ^{yx} is the tangential tension in the normal plain a y and in the direction x, σ is the surface tension, μ is the dynamic viscosity and h(x,t) is the thickness of the liquid film. These a-dimensional groups present also the flow per length unit Q, besides the disturbance parameter ε. This is defined as:

$$\varepsilon = \frac{h_N}{L}; \quad h_N = \left(\frac{3Q\nu}{g}\right)^{1/3} \tag{5}$$

Where h_N is the Nusselt thickness. The asterisk sign characterizes the terms in the dimensional form.

The system of a-dimensionalized equations that reigns the problem's physical behavior is the following (Ataídes, 1994 e Bachir, 1987).

a) Movement Equations (Navier-Stokes):

$$\frac{\partial^2 u}{\partial y^2} + 1 - \frac{\partial P}{\partial x} - \varepsilon\,\mathrm{Re}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + $$
$$+\varepsilon^2\frac{\partial^2 u}{\partial x^2} = 0 \tag{6}$$

$$\frac{\partial P}{\partial y} + \varepsilon^2\frac{\partial^2 v}{\partial y^2} + \varepsilon^3\,\mathrm{Re}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) +$$
$$+\varepsilon^4\frac{\partial^2 v}{\partial x^2} = 0 \tag{7}$$

b) Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

c) Condition of the cinematic boundary in the gas-liquid interface

$$v = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} \tag{9}$$

d) Normal Tension in the gas-liquid interface

$$(az)^2\left[P^2 - \left(\sigma\frac{\partial^2 h}{\partial x^2}\right)^2\right] + \varepsilon\{-2\,a\,z[zP^2 +$$
$$+(P_G - a^2\tau_G^{yy})P - z\left(\sigma\frac{\partial^2 h}{\partial x^2}\right)^2]\} +$$
$$+\varepsilon^2\{z^2[(a\frac{\partial h}{\partial x})^2 + 1]P^2 + 2z(P_G - 3a^2\tau_G^{yy} -$$
$$-2a^2 z\frac{\partial v}{\partial y})P + (P_G - a^2\tau_G^{yy})^2 -$$
$$-\left(z\sigma\frac{\partial^2 h}{\partial x^2}\right)^2\left[1 - 2\left(a\frac{\partial h}{\partial x}\right)^2\right]\} = 0 \tag{10}$$

$$\text{Where, } z = \frac{\mu\left(\frac{g^2 Q}{\nu^2}\right)^{1/3}}{\mu_G\left(\frac{g^2 Q}{\nu^2}\right)_G^{1/3}} \tag{11}$$

d) Tangential Tension in the gas-liquid interface

$$\tau_G^{yx} - z\frac{\partial u}{\partial y} - \varepsilon[2a\tau_G^{xx}\frac{\partial h}{\partial x}] +$$
$$+\varepsilon^2[(z\frac{\partial u}{\partial y} - \tau_G^{yx})(\frac{\partial h}{\partial x})^2 - z\frac{\partial v}{\partial x} +$$
$$2(\tau_G^{xx} + 2z\frac{\partial u}{\partial x})\frac{\partial h}{\partial x}] + \varepsilon^4[z\frac{\partial v}{\partial x}(\frac{\partial h}{\partial x})^2] = 0 \tag{12}$$

f) Velocity of the liquid at the wall

$$u = v = 0 \tag{13}$$

In these conservation equations appear, beyond the already cited variables, the normal tensions τ_G^{xx} and τ_G^{yy} that will be disregarded as being insignificant when compared to pressure [4].

Also is noticed the presence of the Reynolds number of the liquid (Re=Q/ν) and also of the parameter a = s/L, where s is the width of the canal.

The EDP system will have to be transformed in order to obtain a sole EDP for the thickness of the film in function of the variables x and t. The next step will be to expand the velocities and pressure of the liquid in asymptotic series in relation to the disturbance parameter ε as will be presented as follows:

$$u = u^{(0)} + \varepsilon u^{(1)} + \varepsilon^2 v^{(2)} + \varepsilon^3 v^{(3)} + ... \quad (14)$$

$$v = v^{(0)} + \varepsilon v^{(1)} + \varepsilon^2 v^{(2)} + \varepsilon^3 v^{(3)} + ... \quad (15)$$

$$P = P^{(0)} + \varepsilon P^{(1)} + \varepsilon^2 P^{(2)} + \varepsilon^3 P^{(3)} + ... \quad (16)$$

Through the application of the equations (13) to (15) in the a-dimensionalized relations (6) to (12) are obtained various equation systems referring to the terms of order $O(\varepsilon^0)$, $O(\varepsilon^1)$, etc.. Solving these systems we obtain an highly non-linear EDP for the thickness of the film h(x,t).

The resulting equation for h(x,t), presenting only the terms of $O(\varepsilon^0)$, is the following:

$$\frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} h\left(h + \sigma \frac{\partial^3 h}{\partial x^3} h + \frac{\tau_G{}^{yx}}{z}\right) + \frac{\sigma}{3} \frac{\partial^4 h}{\partial x^4} h^3 + \frac{1}{2z} \frac{\partial \tau_G{}^{yx}}{\partial x} h^2 + O(\varepsilon^1) = 0 \quad (17)$$

Only the solution of the equation composed by first order terms is sufficient for obtaining results extremely near to the real problem (Ataídes, 1994).

Solution of the non-linear EDP for h(x, t) To obtain the solution of the equation (16) it is necessary to do some transformations in itself to make the application of the convenient numerical methods possible.

The spectral method of the collocation of Chebyshev (Canuto, 1988) will be applied in this paper. Its trial functions are defined in the interval [-1,1] which requires transformations in the equation (16), (Ataídes, 1994).

The tangential tension of the gas phase that is presented in the EDP for h(x,t) will be defined through an experimental relation obtained for interfacial sheering tension (Biage, 1989 and Bharatan, 1978). This relation for τ_i, already a-dimensionalized, is given by:

$$\tau_i = \left(\frac{3}{8}\right)^{1/3} \left(fi \text{ Kp Kv } \mathrm{Re}_G{}^2\right) \quad (18)$$

$$fi = A[0.005 + B(1-\alpha_s)^C]$$

Where $A = 1;\ B = 14.6;\ C = 1.87$ for $(1-\alpha_s) < 0.1$

$$\alpha_s \stackrel{\Delta}{=} \frac{A_G}{A}; \qquad \mathrm{Kp} = \frac{\rho_G (s - h_N) g}{\rho h_N g};$$

$$Kv = \frac{\nu_G{}^2}{(s - h_N)^3 g}; \quad \mathrm{Re}_G = \frac{Q_G}{\nu_G}$$

For the case of the reference system shown in figure 1, there is:

$$\tau_G{}^{yx} = -\tau_i \quad (19)$$

From the preceding considerations the equation (16) will be transformed in:

$$\frac{\partial h}{\partial t} + 6\frac{\partial h}{\partial x} h^2 + 16\varepsilon^3 W \frac{\partial^4 h}{\partial x^4} h^3 + 48\varepsilon^3 W \frac{\partial^3 h}{\partial x^3} \frac{\partial h}{\partial x} h^2 - 3 fi \text{ Kp Kv } \mathrm{Re}_G{}^2 \frac{\partial h}{\partial x} h = O(\varepsilon) \quad (20)$$

Where the Weber number (W) is defined as:

$$W = \left(\frac{\sigma}{\rho g h_N{}^2}\right)$$

Stability technique of the permanent solution .

Like described by Stakgold (1979), in the permanent regime, a problem of limit value in the special coordinates is studied. The problem depending on time is of the initial value type and of the limit value type in space. At each time t, the solution h is an element in a Hilbert space H. When t changes the element h = h (t) moves, in the Hilbert space, according to the following evolution equation:

$$\frac{d}{dt} h(x,t) = \mathrm{F}(\mathrm{h}, \mathrm{Re}_G);\ t > 0;\ h_{t=0} = h_0 \quad (21)$$

Where h_0 is the initial value of h and F is a transformation of H inside itself, where the derivatives in relation to special coordinates are contained.

One of the principal questions in relation to Eq. (20) is the one that deals with the stability of permanent regimes. The element h is considered as being an element of the space H that satisfies:

$$F(h_i, \mathrm{Re}_G) = 0 \quad (22)$$

The element h does not depend on t being a solution of the Eq. (20) with initial value h. The element h is considered a permanent solution or equilibrium state that interacts always by the effect of small disturbances that can increase or stay of the same size. The analysis that permits to characterize the effect of these disturbances consists of studying the Eq. (20) with initial value h_0, near h. It is said that h is stable if a $\varepsilon > 0$ exists so that the solution of h(t) of the Eq. (20) satisfies:

$$\lim_{t \to 0} \|h - h_i\| = 0 \qquad \text{when} \quad \|h_0 - h_i\| \le \varepsilon \quad (23)$$

The element h(t) is considered a solution of the Eq. (20) corresponding to the initial value h_0, near h, as following:

$$\boldsymbol{h} = \boldsymbol{h_i} + \varepsilon \tilde{h} \tag{24}$$

Where ε is a small disturbance parameter.
Substituting the Eq. (23) in Eq. (20) it is obtained:

$$\frac{d\tilde{h}}{dt} = \frac{F(h_i + \varepsilon\tilde{h}, Re_G) - F(h_i, Re_G)}{\varepsilon} = \\ = F_h(h_i, Re_G)\tilde{h} + r \tag{25}$$

Where $F_h(h_i, Re_G)$ is the Fréchet derivation of F in function of h_i and of the parameter ε. The parameter r is the superior order error in h:

$$\lim_{\tilde{h}\to 0} \left(\|r\|/\|h\|\right) = 0. \tag{26}$$

As the initial value of $\tilde{h}$ is small, the behavior of the solution of the Eq. (24) will be determined by the linearized equation.

$$\frac{d\tilde{h}}{dt} = F_h(h_i, Re_G)\, h \tag{27}$$

If all solutions of the Eq. (26) diminish exponentially with t and x, obtaining sufficiently small solutions for $\tilde{h}$(o) is wanted.

For a solution in the form:

$$\tilde{h}(x,t) = A\, e^{[i\, k(x-ct)+\beta t]} \tag{28}$$

Where A is an element of H, independent of time and of the coordinate x, applied in the Eq. (27) the result is in the form:

$$F_h(h_i, Re_G) = \beta - ikc \tag{29}$$

Where k is the wave number, c is the propagation speed of an elementary wave and β is the temporal amplification coefficient. In this temporal instability analysis on periodical waves it is convenient to define a complex variable C that establishes a relation between the variation of $\tilde{h}$ with t and with x. This relation is described in the following way:

$$C = -\frac{\left(\frac{\partial\tilde{h}}{\partial t}\right)}{\left(\frac{\partial\tilde{h}}{\partial c}\right)} \tag{30}$$

Where $C = c + i\frac{\beta}{t}$ (31)

For each mode K there are one or various solutions corresponding to C, which real part is the propagation speed and the imaginary part is the amplification factor. A wave with c>0 and k<o is, convectively, unstable.

Application of the technique in the Equations of the problem

Applying Eq. (21) in Eq. (19) it is obtained

$$F(h_i, Re_G) = -6\frac{\partial h_i}{\partial x}h_i^2 - 16\varepsilon^3 W\frac{\partial^4 h_i}{\partial x^4}h_i^3 - \\ - 48\varepsilon^3 W\frac{\partial^3 h_i}{\partial x^3}\frac{\partial h_i}{\partial x}h_i^2 + 3fiKpKvRe_G^2\frac{\partial h_i}{\partial x}h_i = 0 \tag{32}$$

The surrounding conditions at the liquid's entrance are given by

$$h_i = h_N; \frac{\partial h_i}{\partial x} = \frac{\partial^2 h_i}{\partial x^2} = \frac{\partial^3 h_i}{\partial x^3} = \frac{\partial^4 h_i}{\partial x^4} = 0 \tag{33}$$

At the liquid's exit the limit condition is:

$$h_i = h_N; \frac{\partial h_i}{\partial x} = \frac{\partial^2 h_i}{\partial x^2} = \frac{\partial^3 h_i}{\partial x^3} = \frac{\partial^4 h_i}{\partial x^4} = 0 \tag{34}$$

Where h represents the permanent solution of h(x,t), h_N is the Nusselt thickness and Re_G is the Reynolds number of the gas.

The limit conditions exert a strong influence on the flow structure and consequently, on the flooding. However in this paper will only be explored the situation characterized by the equations (32) and (33).

In the set of equations (31), (32) and (33) will be applied the spectral collocation technique of Chebychev for the solution of the problem

SPECTRAL COLLOCATION METHOD OF CHEBYCHEV.

The spectral methods are differentiated not only by the type of method but also by the specific choice of the trial functions. The most versatile set of trial functions is composed of the Chebychev polynomial that are defined in the interval [-1,1] by :

$$T_k(x) = \cos k\theta \tag{35}$$

Where k is entire not negative, x = cos θ and 0≤θ≤π (interval I). Choosing the trial functions

$$\phi_k(x) = T_k(x); \quad k = 0, 1, 2, ..., N \tag{36}$$

The approximated solutions for h and for the derivatives of the inferior order to h have the following representations:

$$h_i(x,t) = \sum_{k=0}^{N} a_k \phi_k(x); \quad \frac{\partial h_i}{\partial x}(x,t) = \sum_{k=0}^{N} b_k \phi_k(x) \tag{37}$$

$$\frac{\partial h_i}{\partial x^2}(x,t) = \sum_{k=0}^{N} c_k \phi_k(x); \quad \frac{\partial^3 h}{\partial x^3}(x,t) = \sum_{k=0}^{N} d_k \phi_k(x) \tag{38}$$

$$\frac{\partial^4 h}{\partial x^4}(x,t) = \sum_{k=0}^{N} e_k \phi_k(x) \tag{39}$$

Where a_k, b_k, c_k, d_k, and e_k are coefficients of the series.

In the version of the collocation of the METHOD of Residual weights the test functions are the modified delta of Dirac functions written as follows:

$$\Psi_j(x) = \delta(x - x_j), \quad j = 1, 2, ..., N-1 \tag{40}$$

Where the x_j are the collocation points, distinct in the interval [-1,1]. The application of MWR on any standard differential operator designed by M can be written as follows

$$\int_{-1}^{1} M(h_i)\,\Psi_j(x)dx = 0; \quad j = 1, 2, ..., N-1 \tag{41}$$

This reduces to:

$$M(h_i)\big|_{x=x_j} = 0; \qquad j = 1, 2, ..., N-1 \tag{42}$$

The discrete collocation points are defined as

$$x_j = \cos\frac{\pi j}{N}; \quad j = 0, 1, 2, ..., N. \tag{43}$$

The Chebychev polynomials present the following property:

$$T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x) \quad \text{para } k > 1 \tag{44}$$

From where is obtained

$$\left(\frac{1}{k+1}\right)T'_{k+1}(x) + \left(1 - \frac{k}{k-1}\right)T'_{k-1}(x) == 2T_k(x) \tag{45}$$

para k ≥ 3.

The subscript in the Chebychev polynomial represents its derivation in relation to x.

Applying these relations in Eqs. (31), (32) and (33) gets:

$$2ka_k = b_{k-1} - E_k b_{k+1} \tag{46}$$

$$2kb_k = c_{k-1} - E_k c_{k+1} \tag{47}$$

$$2kc_k = d_{k-1} - E_k d_{k+1} \tag{48}$$

$$2kd_k = e_{k-1} - E_k e_{k+1} \tag{49}$$

$$\text{Where} \quad k = 1, 2, ..., N-1; \quad E_k = \begin{matrix} \frac{1}{2} \text{ if } k = 1 \\ 4 \text{ if } k = 2 \\ 1 \text{ other cases} \end{matrix}$$

This whole theory, referring to the spectral method of the collocation of Chebyshev, will make it possible to mount an equation system and calculate the average thickness of the liquid film.

DISCUSSION AND RESULTS

In the numerical implementation of this problem the thickness h(x,t) as well as its derivatives in relation to x will be expanded until the order of 30 terms.

The processing of the execution of the program, to get the flooding point, is the one where the flow of the liquid per unit of area is fixed and the gas flow (Reynolds number) is gradually increased until passing the transition point of the flow. This point is obtained when an important increase in average thickness occurs (in relation to x) followed by a brusque drop of the same. From this drop is concluded that part of the descending liquid is dragged due to interfacial friction provoked by the gas flow. This point, where the first partial drag of the descending liquid occurs, is taken as the flooding point (Biage, 1989).

The occurrence of new peaks in the average thickness of the liquid film characterizes new flooding points until the transition process reaches its final state, when the film will be co-current with the gas (Ataídes, 1994).

The results that will be presented in the next item involve the surface velocities of the gas (J_G) and the liquid (J_l) that are classically used in biphasic flows. These units are defined as

$$J_G = \frac{Q_{VG}}{A_t}; \qquad J_l = \frac{Q_{Vl}}{A_t} \tag{50}$$

Where Q_{VG} and Q_{Vl}, respectively, are volumetric flows of the gas and liquid A_t represents an area of the transversal section of the canal.

Figures 2 and 3 show an evolution of the thicknesses of the liquid film in relation to the length x, considering various Reynolds numbers of the gas, for two different velocities of the liquid. It can be seen that the thickness h(x,t) oscillates along the length of the canal. This oscillation increases with the decrease of the liquid flow. The symmetry presented by the curves is due to the fact that the contour conditions are symmetrical.

It is seen too, in the figures 2 and 3 that the fluctuations and the average thickness of the liquid film increase with the growth of the Reynolds number of the gas. These fluctuations are more significant for smaller liquid flows. This is due to the fact that the wall and interface tensions act in a percentually bigger range inside the liquid film when its thickness is smaller. The fluctuations of the film are essentially due to the viscosity and therefore this is all coherent with the physical behavior of the problem.

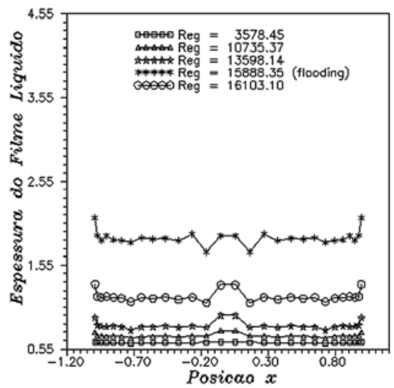

Figure 2. A-dimensional thickness of the liquid film in function of the position x for J_l =0.0088 m/s.

Figure 4 shows the average thickness of the liquid film (in relation to x) versus the Reynolds number of the gas, for three different velocities of the liquid. It can be confirmed that the thickness of the film drops more accentuated when the liquid flow is smaller.

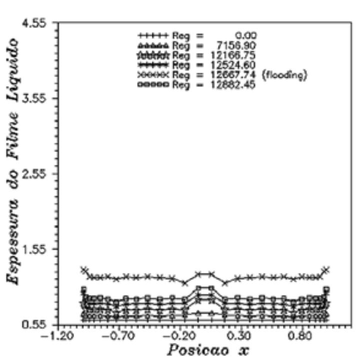

Figure 3. Adimensional thickness of the liquid film in function of the position x for J_l =0.0067 m/s.

Another important fact is that the flooding point increases with the decrease of the liquid flow. Physically this is due to the interactions of the wall and interface tensions. The new peaks after the flooding point, that characterize new liquid drags by the gas, can be observed.

Figure 5 shows the evolution of the average thickness of the liquid film (dimensional) in function of the surface velocity of the gas for a specific surface velocity of the liquid (J_l= 0.035 m/s). The peak of the thickness of the film obtained in this study presents excellent concordance with the one obtained experimentally.

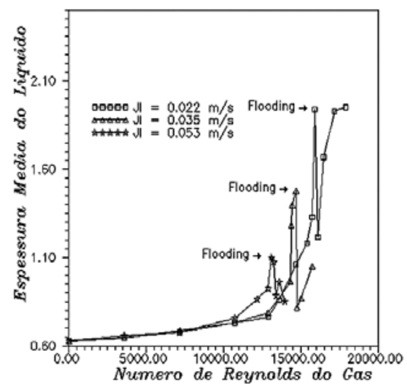

Figure 4. Evolution of the average thickness of the liquid film in relation to x, in function of the Reynolds number of the gas for three surface velocities of the liquid.

The average values, however, show differences of approximately 35% corresponding with less than 0.02 mm. However experimental measurements in canals with a inferior precision become difficult [2].

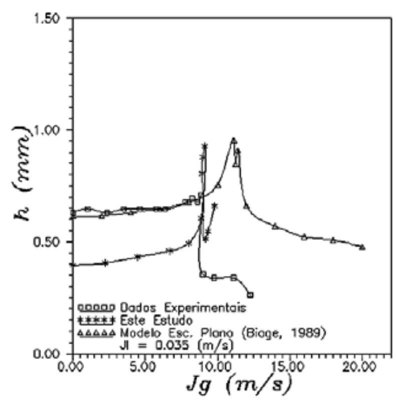

Figure 5. Comparison of the thickness of the liquid film obtained in this paper with the ones from two other studies.

Figure 6 presents the Flooding Chart which axes represent the surface velocity of the liquid versus the surface velocity of the gas. In this figure are plotted, beyond the relation obtained in this paper, the experimental points obtained by Biage (1989), as well as two other experimental relations (Ataídes, 1994 e Biage, 1989). An excellent concordance between the flooding curve referring to this study and that from the experimental results can be observed.

The relative average error confirmed between the values referring to this study and the experimental ones by Biage (1989) is approximately 3%. The studies of Feind (1960) and also of English et al (1963) present relative average errors over 10%.

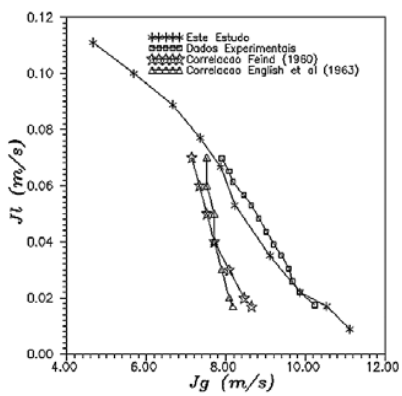

Figure 6. Flooding Chart.

It is concluded that the fluctuating terms that weren't treated in this paper influenced very little in the flow transition leaving on the account of the permanent part the biggest part of the contribution to the phenomenon.

CONCLUSIONS

In this paper is studied the flow of a liquid film on a vertical flat wall submitted to a counter current gas flow. Through a criterion mathematical development was the spectral method of the collocation applied considering only the permanent part of the solution of the thickness of the liquid film obtaining good results. These results were plotted in the flooding chart. It was seen that through this model reliable enough results are obtained and therefore very near the ones obtained experimentally.

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