CONSTRUCTAL DESIGN APPLIED TO THE STUDY OF CAVITIES INTO A SOLID CONDUCTING WALL

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ABSTRACT

This paper relies on the Constructal Design to optimize the geometry of a cavity that penetrates into a solid conducting wall. The objective is to minimize the global thermal resistance between the solid and the cavity. There is uniform heat generation on the solid wall. We studied three shapes of the cavity: rectangular, triangular, and elliptical. The total volume and the cavity volume are fixed with variable aspect ratios. The cavity shape is optimal when it penetrates the conducting wall completely. The rectangular cavity performs better than the elliptical and triangular ones. On the other side, the elliptical cavity has better performance than the triangular one. We also optimized a first construct, i.e., a cavity shaped as T. The performance of the T-shaped cavity is superior to that of the rectangular cavity optimized in the first part of the paper.

Keywords: constructal design, cavities, geometric optimization, heat conduction.

NOMENCLATURE

A	area, m
D_0, D_1	thickness, m
H, L, W	external dimension, m
H_0, L_0	rectangular cavity dimensions, m
k	thermal conductivity, W/m K
q'''	heat generation, W/m^3
T	Temperature, K
V	total volume, m^3
V_0	cavity volume, m^3
x,y	Cartesian coordinates, m

Greek symbols

ϕ	cavity volume fraction

Subscripts

min	minimal
max	maximal

Symbols

~	dimensionless variables

INTRODUCTION

Geometry matters. This is the lesson we have learned from Bejan's Constructal Theory (Bejan, 2000). The principle is the same in engineering and nature: the optimization of flow systems subjected to constraints generates shape and structure (Rosa et al., 2004).

The field of heat transfer has demonstrated for many years how the principle of generating flow geometry works. The oldest and most clear illustration is the optimization of fins. Recent work on this subject account the optimization of the architecture of assemblies of fins (Kraus et al., 2001). Many more examples are found in the growing volume of techniques for the cooling of compact and miniaturized packages of electronics (Peterson and Ortega, 1990, Kim and Lee, 1995, and Bar-Cohen and Kraus, 1998).

This paper proposes the study of the constructal design of another, equally basic feature of a solid wall with heat transfer: the open cavity. Open cavities, for example, are the regions formed between adjacent fins. They are essential promoters of nucleate boiling and condensation: see, for example, the vapotron effect (Biserni et al., 2001, and Falter and Thompson, 1996). Open cavities are also important morphological features in physiology (Bejan, 2000).

We consider the optimization of the cavity shape in the most fundamental sense, without application to a particular device or field. We rely on the constructal method (Bejan, 2000, Biserni et al., 2004, and Rocha et al., 2004): the cavity shape is free to change subject to volume constraints, and in the pursuit of maximal global performance. The global performance indicator is the overall thermal resistance between the volume of the entire system (cavity and solid) and the surroundings.

We start considering two-dimensional geometries where the overall volume and the cavity volume are rectangles with variable geometric aspect ratios (Biserni et al., 2004). Next, we explore elliptical and triangular cavities and compare the performance of these shapes of cavity with the rectangular one in the same basis. Finally, we investigate the simplest first construct: the T-shaped cavity.

RECTANGULAR CAVITY

Consider the two-dimensional conducting body shown in Fig. (1). The external dimensions (H, L) vary. The third dimension, W, is perpendicular to the plane of the figure. The total volume occupied by this body is fixed,

$$V = HLW \tag{1}$$

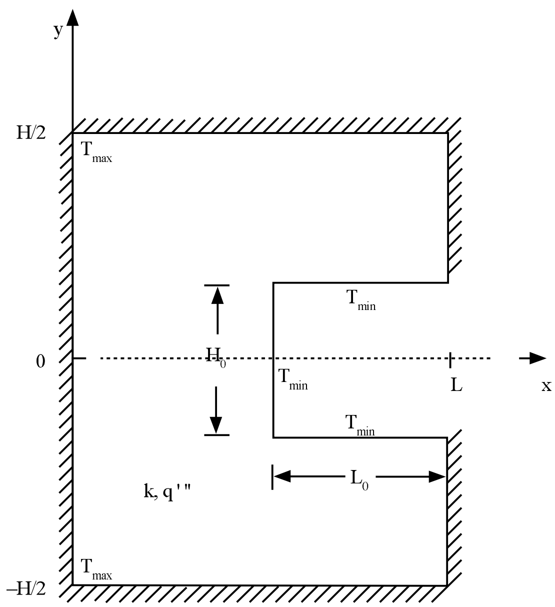

Figure 1. Isothermal rectangular cavity into a two-dimensional conducting body with uniform heat generation.

Alternatively, the area A = HL is fixed. The dimensions of the rectangular cavity (H_0, L_0) also vary. The rectangular cavity volume is fixed,

$$V_0 = H_0 L_0 W \tag{2}$$

The constant volume constraint is justified in many applications: the cost of material and the weight and space of the heat transfer device make this constraint indispensable in design work. In constructal design, this constraint is part of the mechanism of generating the optimal geometric form that fills a given space. This constraint may be replaced by the statement that the volume fraction occupied by the cavity is fixed,

$$\phi = \frac{V_0}{V} = \frac{H_0 L_0}{HL} \tag{3}$$

The solid is isotropic with the constant thermal conductivity k. It generates heat uniformly at the volumetric rate q'''[W/m^3]. The outer surfaces of the heat generating body are perfectly insulated. The generated heat current (q'''A) is removed by cooling the wall of the cavity. The cavity wall temperature is maintained at T_{min}. Temperatures in the solid are higher than T_{min}. The highest temperatures (the "hot spots") are registered at points on the adiabatic perimeter, for example, in the two corners indicated with T_{max} in Fig. (1).

The isothermal cavity wall assumption is made for simplicity in demonstrating the construction of optimal cavity shape. This assumption means that the heat transfer coefficient on the internal (exposed) surface of the cavity is sufficiently large, so that wall conduction poses a larger thermal resistance than convection. This assumption can be relaxed in future applications of this constructal method. In a more realistic model, the cavity wall temperature and heat flux would be related, and the wall temperature distribution would vary with the shape of the cavity.

An important thermal design constraint is the requirement that temperatures must not exceed a certain level. This makes T_{max} a constraint. The design also calls for installing a maximum of heat generation rate in the fixed volume, which, for example, corresponds to packing the most electronics into a device of fixed size. In the present problem statement, this design objective is represented by the maximization of the global thermal conductance q'''A/(T_{max} – T_{min}), or by the minimization of the global thermal resistance (T_{max} – T_{min})/(q'''A).

The numerical optimization of the geometry consisted of simulating the temperature field in a large number of configurations, calculating the global thermal resistance for each configuration, and selecting the configuration with the smallest global resistance. The conduction equation for the solid region is

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} + 1 = 0 \tag{4}$$

where the dimensionless variables are

$$\tilde{T} = \frac{T - T_{min}}{q''' A / k} \tag{5}$$

$$\left(\tilde{x}, \tilde{y}, \tilde{H}, \tilde{L}, \tilde{H}_0, \tilde{L}_0\right) = \frac{(x, y, H, L, H_0, L_0)}{A^{1/2}} \tag{6}$$

The boundary conditions are indicated in Fig. (1). The maximal excess temperature, $\tilde{T}_{max}$, is also the dimensionless global thermal resistance of the construct,

$$\tilde{T}_{max} = \frac{T_{max} - T_{min}}{q''' A / k} \tag{7}$$

The global objective function $\tilde{T}_{max}$ can be determined numerically, by solving for the temperature field in every assumed configuration, and then calculating $\tilde{T}_{max}$ to see whether $\tilde{T}_{max}$ can be minimized by varying the configuration.

Equation (4) was solved using the MATLAB partial-differential-equations (pde) toolbox (MATLAB, 2000). The grid refinements to achieve grid independence and the accuracy of the numerical results can be learned in Cesare et al., (2004). Figure (2) shows that the thermal resistance can be minimized by selecting the shape of the cavity.

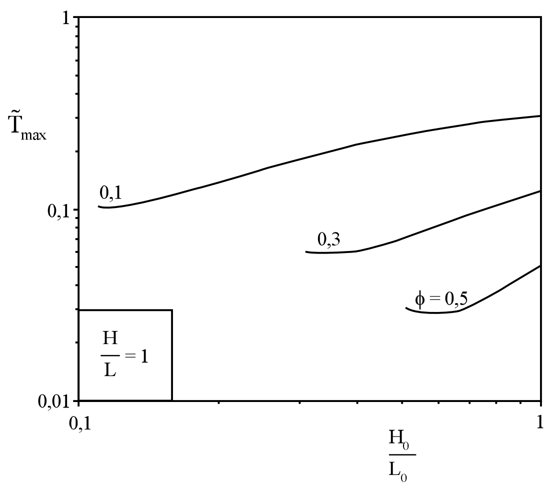

Figure 2. The minimization of the global thermal resistance when the external shape of the heat generation body is fixed and the cavity is rectangular.

The thermal resistance decreases when the volume fraction (ϕ) occupied by the cavity increases.

ELLIPTICAL CAVITY

The elliptical cavity domain is shown in Fig. (3). We kept the two global constraints: the total volume of the system, Eq. (1), and the total volume of the cavity

$$V_0 = \pi H_0 L_0 W / 4 \tag{8}$$

The volume fraction of the volume occupied by the elliptical cavity is given by

$$\phi = V_0 / V = \pi H_0 L_0 / 4HL \tag{9}$$

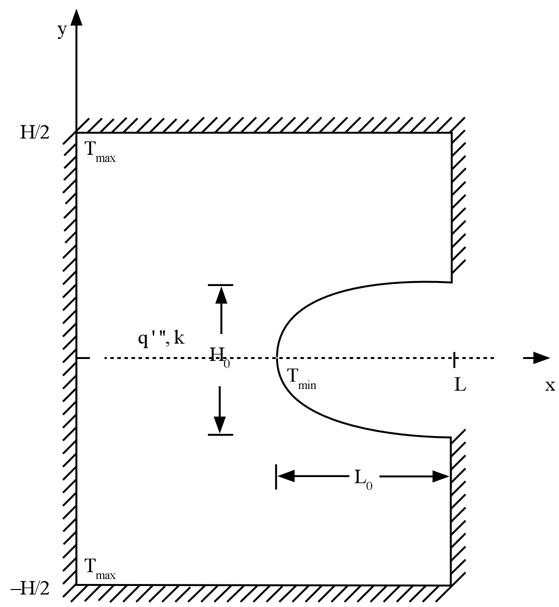

Figure 3. Isothermal elliptical cavity into a two-dimensional conducting body with uniform heat generation.

The heat transfer conduction mathematical model for the points inside the region of conductivity k showed in Fig. (3) is given by Eqs. (4-6). The boundary conditions and the objective function, Eq. (7), are the same used in Section 2. Figure (4) shows the minimization of the thermal resistance with respect to the cavity aspect ratio for several values of the volume fraction (ϕ) using a square heat generation body.

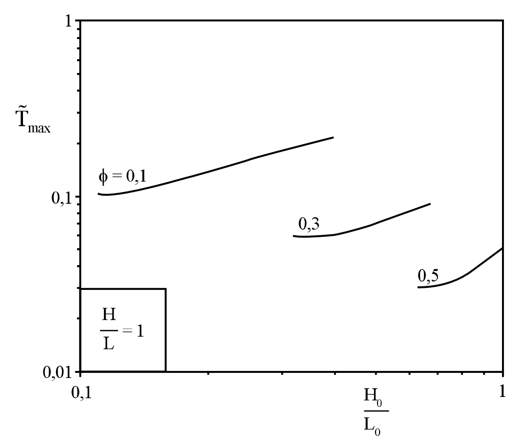

Figure 4. The minimization of the global thermal resistance when the external shape of the heat generation body is fixed and the cavity is elliptical.

TRIANGULAR CAVITY

The triangular cavity domain is shown in Fig. (5). We kept the two global constraints: the total volume of the system, Eq. (1), and the total volume of the cavity, given by

$$V_0 = H_0 L_0 W / 2 \qquad (10)$$

The volume fraction of the volume occupied by the cavity is given by

$$\phi = V_0 / V = H_0\, L_0 / 2HL \qquad (11)$$

The heat transfer conduction mathematical model for the points inside the region of conductivity k showed in Fig. (5) is still given by Eq. (4 - 6). The boundary conditions and the objective function, Eq. (7), are the same used in section 2. Figure 6 shows the minimization of the thermal resistance with respect to the cavity aspect ratio for several values of the volume fraction (ϕ) using a square cavity.

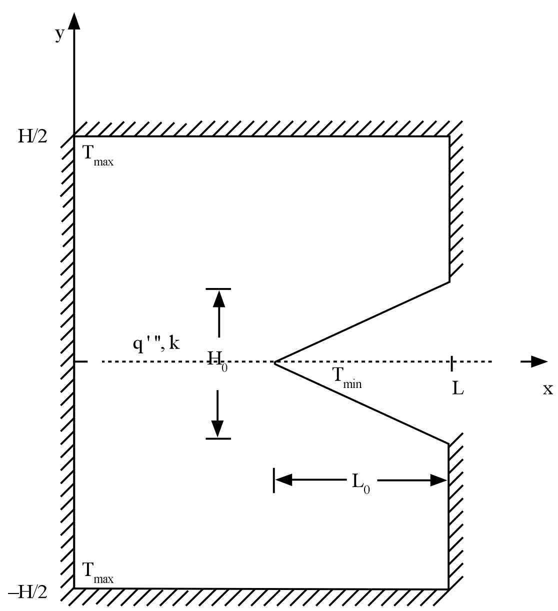

Figure 5. Isothermal triangular cavity into a two-dimensional conducting body with uniform heat generation.

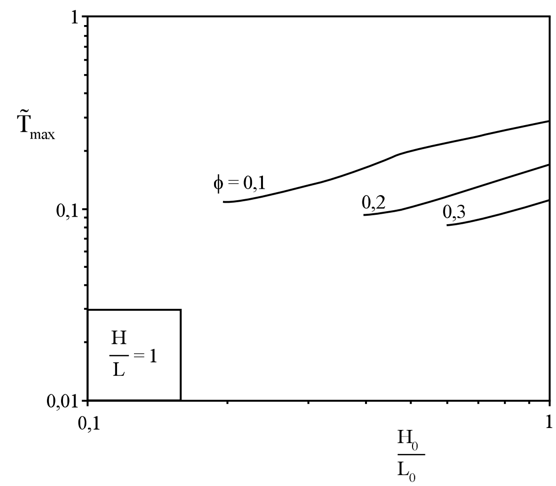

Figure 6. The minimization of the global thermal resistance when the external shape of the heat generation body is fixed and the cavity is triangular.

COMPARISON AMONG THE RECTANGULAR, ELLIPTICAL AND TRIANGULAR CAVITIES

Comparing Figs. (2, 4, and 6), we can learn that the rectangular cavity has better performance than the other ones and the elliptical cavity performs better than the triangular cavity. However, these results are restricted to the case where H/L = 1, i.e., the body is square. To clarify the preceding observation, we conducted a second level of the numerical optimization. The scheme consisted of repeating the preceding work (Figs. (2, 4, and 6)) for many values of the second parameter, H/L. Figure (7) confirms that the rectangular cavity is better than the other ones.

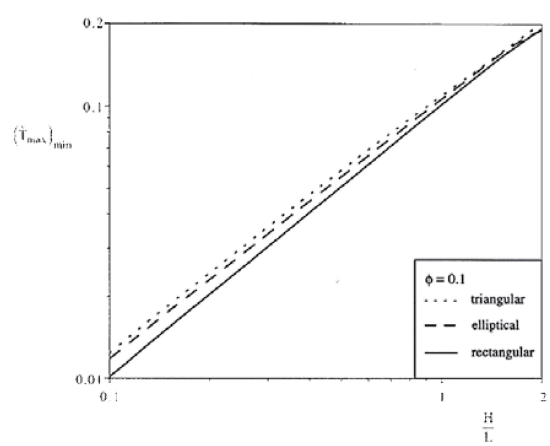

Figure 7. The effect of the external shape H/L on the global thermal resistance minimized as in Figs. 2, 4 and 6.

Figure (7) also shows that an optimal H/L ratio does not exist, i.e., the geometry of Figs. (1, 3, and 5) can be optimized only with respect to one degree of freedom. This means that a second optimization opportunity does not exist because the global resistance minimized with respect to H_0/L_0 varies monotonically with H/L. Figure 8 shows that the cavities reach their best shape when they penetrate the body almost completely, i. e., $L \equiv L_0$.

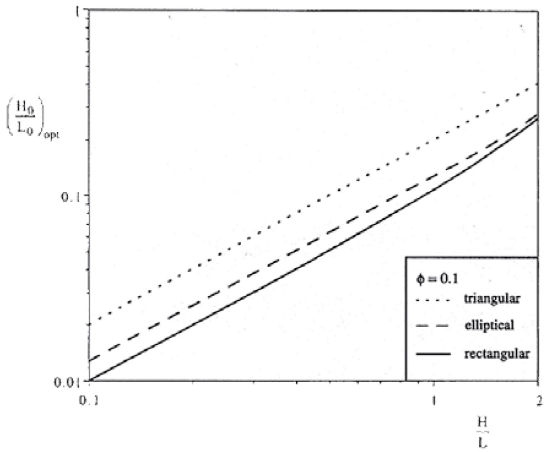

Figure 8. The optimal shape of the cavity as function of H/L.

FIRST CONSTRUCT

The preceding section showed us that the rectangular cavity performs its cooling job better. Constructal design (Bejan, 2000) teaches us that one direction in which the optimization of intrusion geometry can be pursued is that of increasing the complexity of the growing structures, in spaces that grow as they are cooled by expanding flow structures. When the heat generated by the volume is removed through one port, and when the smallest volume element size is fixed, the optimization of geometry generates a tree-shaped flow structure (Bejan, 2000). The simplest tree-shaped structure is a 'first construct', or an optimized assembly of elemental volumes. Considering the elemental rectangular cavity, the simplest first construct is the T-shaped tree (Bejan et al., 2002).

Figure 9 shows the T-shaped intrusion formed by a 'stem' intrusion ($L_1 \times D_1$) that branches into two elemental intrusions ($L_0 \times D_0$).

The global size of the construct (HL = A) is fixed in Fig. (9). The solid material has the conductivity k and generates heat volumetrically at the uniform rate q'''. The perimeter of the HL rectangle is insulated. The surface of the cavity is isothermal at T_{min}. The hot spot of temperature T_{max} occurs at one or more points in the solid. The objective continues to be the minimization of the global thermal resistance $\tilde{T}_{max}$, which is defined in the same way shown in Eq. (7).

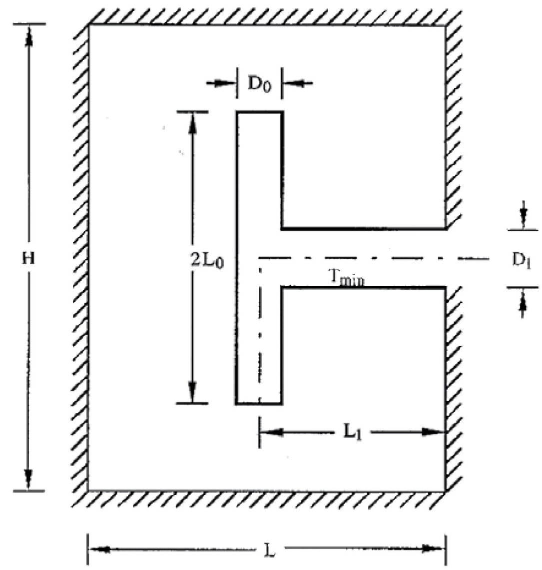

Figure 9. First construct of rectangular cavities arranged as T.

The geometry of the T-shaped construct is subjected to two constraints, the volume fraction occupied by the cavity

$$\phi = \frac{2L_0D_0 + \left(L_1 - D_0/2\right)D_1}{HL} \tag{12}$$

and the volume fraction occupied by the rectangle defined by the 'stem' intrusion,

$$\phi_1 = D_1L_1 / HL \tag{13}$$

The structure has three degrees of freedom, which are represented by the ratios H/L, L_0/L_1 and D_0/D_1. In the following sequence of optimization of results, we fixed H/L = 1, optimized L_0/L_1 by minimizing $\tilde{T}_{max}$ considering a fixed value of D_0/D_1, and then repeated the optimization for other values of D_0/D_1.

Figure 10 shows that there is an optimal D_0/D_1 when ϕ_1 is fixed and the global thermal resistance is minimized with respect to L_0/L_1. We named these results $(D_0/D_1)_{opt}$, $(L_0/L_1)_{opt}$ and $(\tilde{T}_{max})_{min}$.

The procedure used in Fig. (10) is repeated in Fig. (11) where the results using several values of ϕ_1 were collected. Figure (11) shows that the $(D_0/D_1)_{opt}$ ratio decreases when ϕ_1 increases while $(L_0/L_1)_{opt}$ is approximately insensitive to changes in ϕ_1. Figure (11) also shows that the global thermal resistance decreases as ϕ_1 increases. It is interesting to note that all the minimized global thermal resistance values calculated in Fig. (11) are at least 25% smaller than the values of the minimized global thermal resistance showed in Fig. (2) under the same thermal and volume fraction conditions. These results were expected because the T-shaped cavity has more degrees of freedom than the rectangular cavity showed in Fig. (1), i. e., it has more freedom to vary its geometry. The values of the global thermal resistance calculated in Fig. (11) agree well with the results obtained by Biserni et al., (2004) using a different constraint, i. e., instead of ϕ_1 Biserni et al. (2004) used the area defined by the "T", $[2L_0 (L_1 + D_0 / 2) / HL]$, as a constraint.

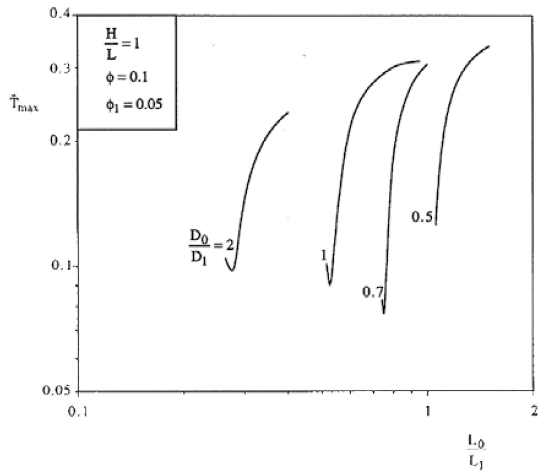

Figure 10. The effect of the ratio L_0/L_1 on the global thermal resistance and on the D_0/D_1 ratio.

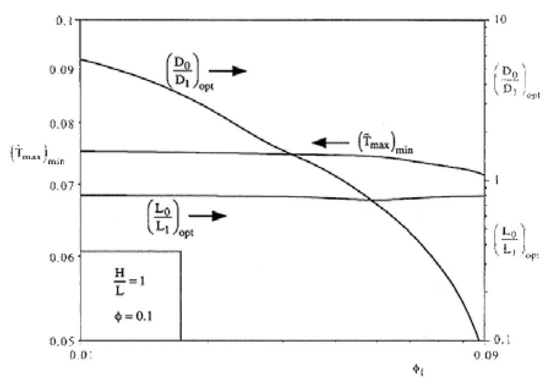

Figure 11. The behavior of the minimized global thermal resistance and the optimized geometry as the constraint ϕ_1 varies.

CONCLUSIONS

Several conclusions and ideas for future research emerged from this study. The rectangular cavity performs better than the elliptical and triangular ones. The elliptical cavity has better performance than the triangular one. All of the studied cavities reach their best shape when they penetrate the body almost completely. This result is independent of the shape of the cavity.

Since we assume isothermal cavity, future work may extend this investigation to the case where the heat transfer on the internal surface of the cavity is accounted for by a constant heat transfer coefficient.

We compare the performance of the elemental volume showed in Fig. (1) with the performance of the first construct drawn in Fig. (9) under the same thermal conditions, uniform heat generation, and the same volume fraction occupied by the cavity, ϕ. Figure (11) shows that, when compared to the rectangular cavity, the T-shaped configuration performs better. This performance improves when ϕ_1 increases. All the minimized global thermal resistance values calculated in Fig. (11) are at least 25% smaller than the values of the minimized global thermal resistance showed in Fig. (2). The T-shaped cavity, the more complex one, has more degrees of freedom. Actually, it has more freedom to vary its geometry. The minimized global thermal resistance calculated for the best T-shaped cavity is 29,4 % smaller than the minimized one shown in Fig. (2). These results agree well with the ones calculated by Biserni et al., (2004).

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REFERENCES

Bar-Cohen, A. and Kraus, A. D., 1998, Advances in Thermal Modeling of Electronic Components and Systems, Vol. 4, ASME Press Series.

Bejan, A., 2000, Shape and Structure, from Engineering to Nature, Cambridge University Press, Cambridge, UK.

Bejan, A., Rocha, L. A. O. and Lorente, S., 2002, Thermodynamic Optimization of Geometry: T- and Y-Shaped Constructs of Fluid Streams, Int. J. Thermal Sciences, Vol. 39, pp. 949-960.

Biserni, C., Lorenzini, G. and Orlandelli, C. M., 2001, Frequency Analysis of the Vapotron Effect in Forced Convection: a Challenge for Numerical Description of Multiphase Flows. In: First Int. Conf. Computational Methods in Multiphase Flow, Multiphase Flow, Orlando, FL, March 2001, WIT Press, pp. 173-182.

Biserni, C., Rocha, L. A. O. and Bejan, A., 2004, Inverted Fins: Geometric Optimisation of the Intrusion into a Conducting Wall, Int. J. Heat Mass Transfer, Vol. 47, pp. 2577-2586.

Falter, H. D. and Thompson, E., 1996, Performance of Hypervapotron Beam-Stopping Elements at Jet, Fusion Technology, Vol. 29, pp. 584-594.

Kim, S. J. and Lee, S. W., 1995, Air Cooling Technology for Electronic Equipment, CRC Press, Boca Raton, FL.

Kraus, A. D., Aziz, A. and Welty, J., 2001, Extended Surface Heat Transfer, Wiley, New York.

MATLAB, User's Guide, Version 6.0.088, release 12, 2000, The Mathworks, Inc.

Peterson, G. P. and Ortega, A., 1990, Thermal control of electronic equipment and devices, Adv. Heat Transfer, Vol. 20, pp. 181-314.

Rocha, L. A. O., Lorenzini, E., and Biserni, C., 2004, Geometric Optimization of Shapes on the Basis of Bejan's Constructal Theory, International Communications Heat and Mass Transfer, Vol. 32, No. 10, pp. 1281-1288.

Rosa, R. N., Reis, A. H., and Miguel A. F., 2004, Bejan's Constructal theory of Shape and Structure, Evora Geophysics Center, University of Evora, Portugal.

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