HEAT TRANSFER STUDY IN SLUG FLOW ON ELLIPTICAL DUCTS CROSS SECTIONS BY GENERALIZED INTEGRAL TRANSFORM TECHNIQUE

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ABSTRACT

This work shows the calculation of heat transfer parameters for slug flow in the thermal entrance region of elliptical section tubes submitted to a second kind boundary condition. The main difficulty in the application of the boundary conditions in problems with this kind of geometry has been removed by using a suitable coordinate change. The generalized integral transform technique (GITT) has been used to obtain the solution of the energy equation. The mixture temperature and the local and average Nusselt numbers have been calculated for several aspect ratios and the results have been compared with those found in the literature.

Keywords: Forced convection, slug flow, integral transform, mixture temperature, elliptical ducts.

NOMENCLATURE

- $a$: major ellipsis semi-axis, m
- $A_y$: non transformed coefficient
- $A_{ac}$: area of the elliptical section, m$^2$
- $b$: minor ellipsis semi-axis, m
- $B_{tmo}$: equation coefficients
- $c_p$: specific heat, J/kg.K
- $C_{t}$: transformed coefficient
- $D_{tm}$: transformed coefficient
- $D_h$: hydraulic diameter, m
- $G$: equation coefficient
- $h$: heat transfer coefficient, W/m$^2$.K
- $h_w$: metric coefficient
- $h_e$: metric coefficient
- $H$: equation coefficient
- $J$: Jacobian determinant
- $k$: fluid conductivity, W/m.K
- $K$: normalized eigenfunction
- $L_{th}$: thermal entry length
- $M$: normalization integral
- $N$: normalization integral
- $Nu$: local Nusselt number
- $Nu_{av}$: average Nusselt number
- $P$: perimeter of the elliptical contour, m
- $Pe$: Peclet number
- $Pr$: Prandtl number
- $q_0^*$: heat flux at the contour, W/m$^2$
- $Re$: Reynolds number
- $T$: temperature, K
- $T_{av}$: average temperature, K
- $T_{w,av}$: average wall temperature, K
- $T_0$: constant inlet temperature, K
- $u, v$: dimensionless elliptical coordinates
- $u_{av}$: specific internal energy, kJ/kg
- $v_0$: coordinate contour
- $w$: velocity field, m/s
- $w_0$: velocity profile for fully developed laminar flow, m/s
- $x, y, z$: coordinate axis, m
- $X, Y, Z$: dimensionless coordinate
- $Z_m$: normalized eigenfunction

Greek symbols

- $\alpha$: dimensionless ellipsis semi-axis
- $\alpha^*$: dimensionless focal distance
- $\beta$: dimensionless ellipsis semi-axis
- $\Gamma$: cross section contour
- $\phi$: eigenfunction related to the coordinate $v$
- $\eta$: normal coordinate, m
- $\eta^*$: dimensionless normal coordinate
- $\lambda$: ellipsis semi-axis ratio
- $\lambda_m$: eigenvalue associated to the $\phi_m(v)$
- $\mu$: eigenvalue associated to the $\phi_1(u)$
- $\theta$: dimensionless temperature
- $\theta_{av}$: dimensionless average temperature
- $\theta_{w,av}$: dimensionless average wall temperature
- $\theta^*$: dimensionless temperature
- $\tilde{\theta}$: transformed potential in $u$ direction
- $\tilde{\theta}^*$: transformed potential in $v$ direction
- $\rho$: fluid density, kg/m$^3$
- $\rho_{av}$: aspect ratio
- $\Sigma$: cross section domain
- $\psi$: eigenfunction related to the coordinate $u$
**ANALYSIS**

In the formulation of the present problem, thermally and hydrodynamically developing laminar steady state flow has been assumed. The effects of viscous dissipation and axial conduction have been neglected and constant fluid properties have been admitted constant throughout. Therefore, the energy equation becomes:

\[
\rho c_p w(x,y) \frac{\partial T}{\partial z} = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]
\]

(1)

with \( \{(x,y) \in \Omega, \ z > 0 \} \).

The velocity profile for fully developed laminar flow inside elliptical tubes is:

\[
w(x,y) = w_0 = \text{constant} \quad \{(x,y) \in \Omega, \ z > 0 \}
\]

(2)

In this model, the inlet condition is:

\[
T(x,y,z) = T_0 \quad \{(x,y) \in \Omega, \ z > 0 \}
\]

(3)

The condition of constant heat flux at the boundary is given by:

\[
-k \frac{\partial T(x,y,z)}{\partial \eta} = q^* \quad \{(x,y) \in \Gamma, \ z > 0 \}
\]

(4)

Finally, according to the symmetry of the problem:

\[
\frac{\partial T(x,y,z)}{\partial x} \bigg|_{z=0} = 0
\]

(5)

\[
\frac{\partial T(x,y,z)}{\partial y} \bigg|_{y=0} = 0
\]

(6)

**Coordinates Transformation**

The main difficulty related to the boundary conditions application to elliptical geometries has been overcome by employing the coordinate change shown in Fig. 1.
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\[ \alpha = \frac{a}{D_h} \]  
\[ \beta = \frac{b}{D_h} \]  
\[ D_h = \frac{4A_{sc}}{P} \]  
\[ \rho_{wp} = \frac{b}{a} \]  
\[ A_{sc} = \pi a b \]  
\[ P = 4a \int_{0}^{\pi} \sqrt{1 - \kappa^2 \sin^2 \theta} \ d\theta \]  
\[ \kappa = \frac{\sqrt{a^2 - b^2}}{a} \]  

With these new variables, the energy equation becomes:

\[ \frac{\partial \theta(X,Y,Z)}{\partial Z} = \frac{\partial^2 \theta(X,Y,Z)}{\partial X^2} + \frac{\partial^2 \theta(X,Y,Z)}{\partial Y^2} \]  

The boundary conditions and the entrance condition are as follows:

\[ \theta(X,Y,Z) = 0 \quad \{ (x,y) \in \Omega, \ Z = 0 \} \]  
\[ \frac{\partial \theta(X,Y,Z)}{\partial \eta^*} = 1 \quad \{ (x,y) \in \Gamma, \ z > 0 \} \]  
\[ \frac{\partial \theta(X,Y,Z)}{\partial X} = 0 \quad \{ X = 0, \ Z > 0 \} \]  
\[ \frac{\partial \theta(X,Y,Z)}{\partial Y} = 0 \quad \{ Y = 0, \ Z > 0 \} \]  

The orthogonal system of elliptic coordinates \((u, v)\) is used to transform the original domain, with elliptical contour in the coordinates \((X, Y)\), into a domain with rectangular contour in the transformed system \((u, v)\):
\[ X = \alpha^* \cosh(v) \cos(u) \]  
\[ Y = \alpha^* \sinh(v) \sin(u) \]  
\[ Z = z \]  
\[ \alpha^* = \frac{\alpha}{\cosh(v_0)} \]  
\[ v_0 = \text{arctanh} \left( \frac{\beta}{\alpha} \right) \]  
\[ \{ 0 < \beta < \alpha \} \]  

The coefficients \( h_u \) and \( h_v \) and the Jacobian \( J \) of the transformation of the system of coordinates \((X, Y)\) into the system \((u, v)\) are given by:

\[ h_u(u, v) = h_v(u, v) = h(u, v) \]  
\[ h(u, v) = \alpha^2 \sqrt{\sinh^2(v) + \sin^2(u)} \]  
\[ J(u, v) = \frac{\partial(XY)}{\partial(u, v)} = \alpha^2 [\sinh^2(v) + \sin^2(u)] \]  

With these new variables the equations of the elliptical contour and energy become, respectively:

\[ \left[ \frac{X}{\alpha^* \cosh(v_0)} \right]^2 + \left[ \frac{Y}{\alpha^* \sinh(v_0)} \right]^2 = 1 \]  

\[ H(u, v) \frac{\partial \theta(u, v, Z)}{\partial Z} = \frac{\partial^2 \theta(u, v, Z)}{\partial u^2} + \frac{\partial^2 \theta(u, v, Z)}{\partial v^2} \]  

with \( H(u, v) \) given by:

\[ H(u, v) = J(u, v) \]  

The entry and boundary conditions in the new coordinate system become:

\[ \theta(u, v, Z) = 0 \quad \{ (u, v) \in \Omega, \ Z = 0 \} \]  

\[ \frac{\partial \theta(u, v, Z)}{\partial u} = 0 \quad \{ u = 0, \ 0 \leq v \leq v_0, \ Z > 0 \} \]  

\[ \frac{\partial \theta(u, v, Z)}{\partial u} = 0 \quad \{ u = \pi/2, \ 0 \leq v \leq v_0, \ Z > 0 \} \]  

\[ \frac{\partial \theta(u, v, Z)}{\partial v} = 0 \quad \{ 0 \leq u \leq \pi/2, \ v = v_0, \ Z > 0 \} \]  

\[ \frac{\partial \theta(u, v, Z)}{\partial v} = 0 \quad \{ 0 \leq u \leq \pi/2, \ v = v_0, \ Z > 0 \} \]

**Homogenization of the Boundary Conditions**

For the application of the GITT, it is convenient to implement the homogenization of the boundary conditions to increase the convergence rate of the series that represents the solution. In order to accomplish this, the following change of variables is considered:

\[ \theta(u, v, Z) = \theta^*(u, v, Z) + \frac{v^2}{2v_0} h(u, v) \]  

With this new variable, the energy equation is:

\[ H(u, v) \frac{\partial \theta^*(u, v, Z)}{\partial Z} = \frac{\partial^2 \theta^*(u, v, Z)}{\partial u^2} + \frac{\partial^2 \theta^*(u, v, Z)}{\partial v^2} + G(u, v) \]

with:

\[ G(u, v) = \frac{h(u, v)}{v_0^2} + \frac{\alpha^* v^2}{2h(u, v) v_0} \left[ \cos(2u) - \frac{\alpha^*}{2h(u, v) v_0} \sin^2(2u) \right] \]

The entry and boundary conditions are then redefined by:

\[ \theta^*(u, v, Z) = -\frac{v^2}{2v_0} h(u, v) \quad \{ (u, v) \in \Omega, \ Z = 0 \} \]

\[ \frac{\partial \theta^*(u, v, Z)}{\partial u} = 0 \quad \{ u = 0, \ 0 \leq v \leq v_0, \ Z > 0 \} \]

\[ \frac{\partial \theta^*(u, v, Z)}{\partial u} = 0 \quad \{ u = \pi/2, \ 0 \leq v \leq v_0, \ Z > 0 \} \]

\[ \frac{\partial \theta^*(u, v, Z)}{\partial v} = 0 \quad \{ 0 \leq u \leq \pi/2, \ v = v_0, \ Z > 0 \} \]
\[ \frac{\partial \theta^* (u,v,Z)}{\partial v} = 0 \quad \{0 \leq u \leq \pi/2, \ v = 0, \ Z > 0 \} \quad (48) \]

Application of the GITT

To obtain the solution of the diffusion equation in the new coordinate system, Eq. (42), subjected to the conditions given by Eqs. (44)-(48), the Generalized Integral Transform Technique is applied. In order to accomplish this, the following auxiliary eigenvalue problem related to the independent variable \( u \) is considered:

\[ \frac{d^2 \psi(u)}{du^2} + \mu_i^2 \psi(u) = 0 \quad \{0 \leq u \leq \pi/2 \} \quad (49) \]

with the boundary conditions:

\[ \frac{d \psi(u)}{du} = 0 \quad \{u = 0 \} \quad (50) \]

\[ \frac{d \psi(u)}{du} = 0 \quad \{u = \pi/2 \} \quad (51) \]

The eigenvalues and eigenfunctions associated to this problem are:

\[ \mu_i = 2i \quad (i = 1, 2, 3 \ldots) \quad (52) \]

\[ \psi_i (u) = \cos \mu_i u \quad (53) \]

The above eigenfunctions are orthogonal, allowing the following pair inverse-transform:

\[ \tilde{\theta}^*_i (v,Z) = \int_0^{\pi/2} K_i (u) \theta^* (u,v,Z) \, du \quad (54) \]

\[ \theta^* (u,v,Z) = \sum_{i=1}^{\infty} K_i (u) \tilde{\theta}^*_i (v,Z) \quad (55) \]

Where \( K_i (u) \) are the normalized eigenfunctions given by:

\[ K_i (u) = \frac{\psi_i (u)}{N_i^{1/2}} \quad (56) \]

where:

\[ N_i = \int_0^{\pi/2} \psi_i^2 (u) \, du = \begin{cases} \pi/2, & i = 1 \\ \pi/4, & i > 1 \end{cases} \quad (57) \]

According to the formalism presented by Cotta (1998), Eq. (34) and Eq. (42) are multiplied by operators \( \int_0^{\pi/2} K_i (u) \, du \) and \( \int_0^{\pi/2} \theta^* (u,v,Z) \, du \), respectively. Following this procedure and applying the boundary conditions described by Eqs. (37)-(40) and Eqs. (45)-(48), the obtained system is:

\[ \sum_{j=1}^{\infty} A_{ij} \frac{\partial \tilde{\theta}^*_i (v,Z)}{\partial Z} + \mu_i^2 \tilde{\theta}^*_i (v,Z) = \frac{\partial^2 \tilde{\theta}^*_i (v,Z)}{\partial u^2} + C_i (v) \quad (58) \]

where:

\[ A_{ij} (v) = \int_0^{\pi/2} K_i (u) K_j (u) H(u,v) \, du \quad (59) \]

\[ C_i (v) = \int_0^{\pi/2} K_i (u) G(u,v) \, du \quad (60) \]

Let us now consider the following eigenvalue problem related to the independent variable \( v \):

\[ \frac{d^2 \phi(v)}{dv^2} + \lambda^2 \phi(v) = 0 \quad \{0 \leq v \leq v_o \} \quad (61) \]

This problem is subjected to the following boundary conditions:

\[ \frac{d \phi(v)}{dv} = 0 \quad \{v = 0 \} \quad (62) \]

\[ \frac{d \phi(v)}{dv} = 0 \quad \{v = v_o \} \quad (63) \]

The eigenvalues and the eigenfunctions for this new problem are:

\[ \lambda_m = \frac{(m-1)\pi}{v_o} \quad (m = 1, 2, 3 \ldots) \quad (64) \]

\[ \phi_m (v) = \cos (\lambda_m v) \quad (m = 1, 2, 3 \ldots) \quad (65) \]

These eigenfunctions are orthogonal and allow the following pair inverse-transform:
where $Z_m(v)$ are the normalized eigenfunctions:

$$Z_m(v) = \frac{\varphi_m(v)}{M_m}$$

(68)

$$M_m = \int_0^{v_0} \varphi_m^2(v) \, dv = \begin{cases} v_0, & m > 0 \\ v_0/2, & m > 0 \end{cases}$$

(69)

To determine the transformed temperature equation $\tilde{\theta}_m(Z)$, the procedure is similar to that one related to the first eigenvalue problem. Eq. (24) and Eq. (32) are multiplied by operators $\int_0^{v_0} Z_m(v) \, dv$ and $\int_0^{v_0} \tilde{\theta}_m^+(v,Z) \, dv$, respectively, and using boundary conditions Eqs. (37)-(40) and Eqs. (45)-(48), the following system of ordinary differential equations is obtained:

$$\sum_{n=1}^N \sum_{m=1}^M B_{ijmn} \frac{d\tilde{\theta}_m(Z)}{dZ} + (\mu_i^2 + \lambda_n^2) \tilde{\theta}_m(Z) + D_{im} = 0$$

(70)

with:

$$B_{ijmn} = \int_0^{v_0} Z_m(v) Z_n(v) A_i(\nu) A_j(\nu) \, dv =$$

(71)

$$\int_0^{v_0} \tilde{\theta}_m^+(v) K_i(u) K_j(u) Z_m(v) K_n(v) H(u,v) \, du \, dv$$

$$D_{im} = -\int_0^{v_0} Z_m(v) C_i(v) \, dv =$$

$$-\int_0^{v_0} \tilde{\theta}_m^+(v) K_i(u) Z_m(v) G(u,v) \, du \, dv$$

(72)

The parameters $B_{ijmn}$ and $D_{im}$ can be integrated and, therefore, determined. The solution of this system of ordinary differential equations, when subjected to the transformed entry condition, is the following

$$\tilde{\theta}_m(0) = \int_0^{v_0} \tilde{\theta}_m^+(u) Z_m(u) \theta^+(u,v,0) \, du \, dv =$$

$$= -\int_0^{v_0} \tilde{\theta}_m^+(u) \frac{v^2}{2 v_0} h(u,v,0) \, du \, dv$$

(73)

The transformed temperature $\tilde{\theta}_m^+(Z)$ is obtained from Eq. (70). Thus, the dimensionless temperature $\theta(u, v, Z)$ can be numerically evaluated using Eq. (66), just truncating the expansion in series of orthogonal functions, for a given order $i = N$ and $m = M$:

$$\theta(u,v,Z) = \sum_{i=1}^N \sum_{m=1}^M K_i(u) Z_m(v) \tilde{\theta}_m^+(Z) +$$

$$\frac{v^2}{2 v_0} h(u,v,0)$$

(74)

Obviously, larger values of $N$ and $M$ would result in higher accuracy of the numerical results, neglecting round-off errors.

**Average Fluid Temperature and Nusselt Numbers**

The average fluid temperature for a given duct cross section can be determined by means of a balance of energy between the tube inlet section and the given cross section, at a position $z$ along the axis:

$$\dot{q}_0 P z = \rho u \alpha_c c_p (T_{in} - T_0)$$

(75)

The dimensionless average fluid temperature is defined by:

$$\theta_\text{av}(z) = 4 Z$$

(76)

The dimensionless average fluid temperature can also be determined through the integration of the temperature distribution:

$$\theta_\text{av}(Z) = \frac{D^2}{A} \int_\Omega \theta(X,Y,Z) w(X,Y) \, d\Omega$$

(77)

In the plane $(u, v)$, $\theta_\text{av}$ is expressed by the following equation:
\[ \theta_{av}(Z) = \frac{D_h^2}{A} \int_0^{z^2} \left[ \sum_{i=1}^{N} \sum_{m=1}^{M} K_i(v) Z(v) \theta_{im}(Z) \right] \right] + \frac{v}{2y_0} h(u,v) H(u,v) \, du \, dv \] (78)

Equations (76) and (78) are adequate for the verification of the accuracy of the numerical results when the expansion is truncated in orders \( i = N \) and \( m = M \).

The average wall temperature is obtained by the integration:

\[ \theta_{w,av}(Z) = \frac{4D_h}{P} \int_{v_0}^{z^2} \sum_{i=1}^{N} \sum_{m=1}^{M} K_i(v) Z(v) \theta_{im}(Z) \right] \right] \, du \] (79)

The Nusselt number is defined by:

\[ Nu(z) = \frac{h(z) D_h}{k} \] (80)

with \( h(z) \) defined as follows:

\[ h(z) = \frac{q^\circ(z)}{T_{w,av} - T_{av}(z)} \] (81)

By using the dimensionless variables, the Nusselt number becomes:

\[ Nu(Z) = \frac{l}{\theta_{w,av} - \theta_{av}} \] (82)

The average Nusselt number is obtained by integrating Eq. (82) along the tube axis:

\[ Nu_{av}(Z) = \frac{l}{Z} \int_0^Z Nu(Z') \, dZ' \] (83)

Shah and London (1978) define the thermal entry length (\( L_{th} \)) as the position where the local Nusselt number is 5% higher than the Nusselt number in the region where the fluid is thermally developed:

\[ L_{th} = \text{positive root of } \{ l | 0.05 \, Nu(\infty) - Nu(Z) = 0 \} \] (84)

RESULTS AND DISCUSSION

In order to determine the coefficients \( \theta_{im}(Z) \), the expansion given by Eq. (70) has been truncated to several choices of \( M \) and \( N \) values. Parameters \( B_{im} \) and \( D_{im} \) have been numerically calculated by a Gauss quadrature method (36 points of quadrature) and the equation system, Eq. (74), has been solved by using the routine DIVPAG of the IMSL Fortran Numerical Library (IMSL Library, 1979).

It has been noticed that the convergence becomes slower when the aspect ratio \( b/a \rightarrow 1 \). For truncations to \( M \) and \( N \) both higher than 25, the values of calculated Nusselt number, for \( Z > 0.0001 \), converged within around 3 digits or more, for the analyzed cases. Therefore, all calculations in the present study truncated the expansion to \( M = 25 \) and \( N = 25 \), generating a system of 625 ordinary differential equations.

The difference between the average temperature calculated by Eq. (78) and the average temperature calculated by Eq. (76) is less than \( 10^{-2} \) for \( Z > 0.0001 \). The results for average wall temperature, local and average Nusselt numbers are presented in Table 1 for aspect ratio \( b/a = 0.5 \). The behavior of these parameters has been shown in Figs. 2 to 4 for several aspect ratios.

<table>
<thead>
<tr>
<th>( z )</th>
<th>( \theta_{av} )</th>
<th>( \theta_{w,av} )</th>
<th>( Nu_{av} )</th>
<th>( Nu_{av} )</th>
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</table>

From the obtained results it may be observed that the thermal development is slower when the aspect ratio \( b/a \rightarrow 0 \). On the other hand, it has been noticed that the values of wall temperature and Nusselt number, calculated for
several eccentricities of ellipses, approximate asymptotically in the region near tube entry. In the region where the flow is thermally developed a strong dependence of these parameters with the eccentricity of the ellipse has been observed, when the aspect ratio \( b/a < 0.5 \). When the aspect ratio \( b/a > 0.9 \), also in the fully developed region, an asymptotical approximation of these parameters has been noticed. In particular, the asymptotical local Nusselt number is \( \text{Nu} = 8.0 \) for circular cross section in the duct.

![Graph showing fluid average temperature and average wall temperature in the Z-axis, for several aspect ratios.](image)

**Figure 2.** Fluid average temperature and average wall temperature in the \( Z \)-axis, for several aspect ratios

Thermal development occurs farther from the duct entrance when aspect ratio \( b/a \to 0 \) and at the limits of \( b/a \) 1.0 the results for the Nusselt number approximate those obtained for flows in ducts of circular section.

![Graph showing local Nusselt number in the Z-axis, for several aspect ratios.](image)

**Figure 3.** Local Nusselt number in the \( Z \)-axis, for several aspect ratios

Finally, Tab. 2 presents the results obtained in this work, corresponding to Nusselt numbers of thermally developed flows and Fig. 5 shows the limit Nusselt and the thermal entry length, for elliptical ducts cross section flow.

![Graph showing Nusselt number and thermal entry length.](image)

**Figure 4.** Average Nusselt number in the \( Z \)-axis, for several aspect ratios

**Figure 5.** Limit Nusselt and the thermal entry length, for elliptical ducts cross section flow

<table>
<thead>
<tr>
<th>( b/a )</th>
<th>( \text{Nu}_\infty )</th>
<th>( L_{\text{th}} )</th>
</tr>
</thead>
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<td>0.047</td>
</tr>
<tr>
<td>0.99</td>
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</table>
CONCLUSIONS

In the present work, the GITT has been successfully applied to the problem of thermally developing flow inside ducts of elliptical cross section, subjected to second kind boundary conditions. The inherent difficulty of application of the boundary in problems with this geometry has been overcome by means of a change of the cartesian variables to a system of orthogonal elliptic coordinates, that transforms the contour of elliptical section of the duct to a new domain of rectangular shape. The convergence of the temperature distribution is slow and it is necessary to truncate M and N at a relatively high order. For regions near the duct entrance, $Z \approx 0.0001$, it has been necessary to truncate the series to $M = N > 25$ to obtain a satisfactory convergence for the Nusselt number calculations, although for values of $Z$ near the region of thermal development, satisfactory convergence has been obtained with fewer terms in the series. In fact, in the fully developed region, five terms in the series have shown to be usually sufficient for a good numerical convergence.

The results obtained through GITT allowed the determination of the parameters of interest, as well as the wall temperature and the local and average Nusselt number. Moreover, the results for the Nusselt number obtained for the thermally developed region exhibited excellent agreement with those reported in the literature at the limits of $b/a \to 1.0$ (circular section ducts).

REFERENCES


