EVALUATION OF FINITE MIXTURE MODELS FOR DESCRIBING THE STRUCTURE OF DISTURBED GMELINA STANDS IN OLUWA FOREST RESERVE, NIGERIA

Friday Nwabueze Ogana

Department of Social and Environmental Forestry, University of Ibadan, Ibadan, Nigeria; Post code: 200284
fn.ogana@ui.edu.ng; ORCID: 0000-0002-8388-204X

Received for publication: 25/04/2020 - Accepted for publications: 10/11/2020

Resumo

Avaliação de modelos de mistura finita para descrever a estrutura de povoamentos de gmelina perturbados na reserva florestal de Oluwa, Nigéria. O modelo de mistura finita é relevante para descrever a distribuição do diâmetro que é multimodal ou fortemente inclinada. A distribuição irregular do diâmetro é parcialmente causada por distúrbios florestais, como queimadas e extração ilegal de madeira. Este estudo avaliou cinco modelos de mistura finita para descrever as distribuições irregulares do diâmetro dos povoamentos perturbados de *Gmelina arborea* Roxb na Reserva Florestal de Oluwa, Nigéria. Vinte unidades amostrais de tamanho de 0,04 ha, cinco cada em quatro idades de povoamento (19, 24, 29 e 34 anos) foram utilizadas neste estudo. Cinco modelos de mistura finita: gama, Gompertz, log-logístico, lognormal e misturas de Weibull foram considerados. A qualidade dos ajustes produzidos pelos modelos foi avaliada com cinco índices: Anderson-Darling, Cramer-von Mises, Kolmogorov-Smirnov, Akaike Information Criterion e Hannan-Quinn Information Criterion. A soma das classificações relativas dos índices foi analisada por meio da análise de variância de único fator. Os resultados mostraram que a mistura gama teve os menores índices e classificação relativa, mas não significativamente diferente de Weibull e misturas log-logísticas (p> 0,05). As misturas Lognormal e Gompertz tiveram um desempenho ruim. A ordem de classificação foi: gama seguida de Weibull, log-logístico, lognormal e Gompertz. A aplicação dos modelos de mistura proporcionou boas predições do volume do povoamento florestal.

Palavras-chave: Perturbação florestal; gama; Gompertz; log-logistic; lognormal; Weibull

Abstract

Finite mixture model is relevant for describing diameter distribution that is multimodal or heavily skewed. Irregular diameter distribution is partly caused by forest disturbance such as bush burning and illegal logging. This study evaluated five finite mixture models for describing the irregular diameter distributions of the disturbed *Gmelina arborea* Roxb stands in Oluwa Forest Reserve, Nigeria. Twenty plots of 0.04 ha size, five each from four stand ages (19, 24, 29 and 34 years) were used in this study. Five finite mixture models: gamma, Gompertz, log-logistic, lognormal and Weibull mixtures were considered. The quality of fits produced by the models were evaluated with five indices: Anderson-Darling, Cramer-von Mises, Kolmogorov-Smirnov, Akaike Information Criterion and Hannan-Quinn Information Criterion. Relative rank sum from the indices was analysed using One-way analysis of variance. The results showed that gamma mixture had the smallest indices and relative rank, but not significantly different from Weibull and log-logistic mixtures (p > 0.05). Lognormal and Gompertz mixtures performed poorly. The order of ranking was: gamma followed by Weibull, log-logistic, lognormal and Gompertz. Application of the mixture models provided good predictions of the forest stand volume.

Keywords: Forest disturbances; gamma; Gompertz; log-logistic; lognormal; Weibull

INTRODUCTION

Finite mixture model is relevant for describing diameter distribution that is multimodal or heavily skewed (LIU *et al*., 2002; JAWORSKI; PODLASKI, 2012; LIU *et al*., 2014). It has also been used as classification tool in forestry (ZASADA; CIESZEWSKI, 2005). Irregular diameter distribution of forest stands is partly caused by forest disturbances such as wind damage, bush burning, illegal exploitation, thinning and the like (ZHANG; LIU, 2006; TSOGT; LIN, 2012; OGANA *et al*., 2020). These disturbances often create gaps in the forest given rise to heavily skewed, bimodal or multimodal stand structure (PODLASKI, 2017).

In Nigeria, bush burning and uncontrolled exploitation are common forest disturbances affecting both natural and plantation forests (ADEKUNLE, 2006). Because of the continuous disturbance of the forest ecosystem, most forest stand structures, especially those of production forests in Nigeria lack uniformity (single peak). Unfortunately, previous studies have centred on the use of univariate distributions such as the Weibull, Johnson SB, Burr XII, etc., to describe the diameter distributions of production forests in Nigeria (e.g., AJAYI, 2013; EKPA *et al*., 2014; OGANA *et al*., 2017, OGANA; EKPA 2020). Modelling diameter distribution of disturbed forest with a univariate density function often lead to oversimplification of the stand structures (LIU *et al*., 2002).

Recently, Ogana (2018) used mixture models of two components of univariate gamma, lognormal, normal and Weibull distributions to explain the multimodal diameter distribution of degraded *G. arborea* stands in Omo Forest Reserve, Nigeria. The author found this approach to be more effective than single component univariate distribution. Similarly, Ogana *et al*. (2020) recommended the use of finite mixture model for modelling diameter distributions of the *Tectona grandis* Linn f in the same Forest Reserve. They reported better performance with the finite mixture model compared to some single component four-parameter density functions. A good knowledge of the irregular structure of disturbed stands is a prerequisite for its management and the planning of silvicultural treatments. Therefore, the purpose of this study was to evaluate some finite mixture models for describing the diameter distribution of the disturbed *G. arborea* stands in Oluwa Forest Reserve, Nigeria with a view to enhancing its management.

MATERIAL AND METHODS

Data

The data were obtained from the *G. arborea* plantation in Oluwa Forest Reserve, Ondo State, Nigeria. The reserve is situated between Latitude 6°55′ – 7°20′ N and Longitude 3°45′ – 4°32′ E, occupies an area of 87,816 ha and an average elevation of about 123 m above sea level. Oluwa has an average annual temperature of 26℃ and annual rainfall in the range 1700 to 2200 mm (ONYEKWELU *et al*., 2006). Large scale plantation establishment in Oluwa started in the 1960. Over the years, *G. arborea* has emerged as the major plantation species in the reserve, accounting for almost 89% of the total plantation (OGANA; EKPA 2020). The *G. arborea* plantation is a production forest established to supply raw materials for pulp and paper mills (AJAYI, 2013). However, the plantations are now used for timber production due to failure of the mills to exploit them (OGANA *et al*., 2017). Bush burning for the cultivation of agricultural crops by farmers and uncontrolled exploitation of timber are the two anthropogenic activities that have affect the *G. arborea* plantations in the recent times. This has been a major problem of the forestry sector in Nigeria. Data were collected from four age series (19, 24, 29 and 34 years) in the *G. arborea* plantation. Twenty temporary sample plots (TSPs) of 0.04 ha size were demarcated in the *G. arborea* stands using simple random sampling technique. Diameter measurements of all trees at breast height (1.3 m aboveground level, Dbh) were obtained to an accuracy of 0.1 cm with diameter tape. The diameter data were used to compute the quadratic mean diameter (Dg, cm) and the density (number of trees per ha, N). The descriptive statistics for the dataset are presented in Table 1.

Table 1. Descriptive statistics of the *Gmelina arborea* dataset
Tabela 1. Estatística descritiva do conjunto de dados de *Gmelina arborea*

Stands	**Statistics**	**Variables**		
		Dbh (cm)	**Dg (cm)**	**N (trees ha^{-1})**
Age 19	Mean	24.1	25.4	870.0
(190 trees)	SD	10.1	4.23	164.3
	Min	6.6	20.7	625.0
	Max	46.5	31.0	1075.0
Age 24	Mean	23.5	25.5	1150.0
(171 trees)	SD	10.6	1.83	93.5
	Min	4.0	23.2	1075.0
	Max	54.5	27.7	1300.0
Age 29	Mean	22.4	23.8	1435.0
(287 trees)	SD	9.8	1.77	123.2
	Min	5.5	21.5	1225.0
	Max	51.4	26.2	1525.0
Age 34	Mean	19.7	21.9	855.0
(230 trees)	SD	10.2	3.34	290.7
	Min	3.0	17.9	425.0
	Max	46.5	26.1	1175.0

Dbh: diameter at breast height (1.3 m above the ground); Dg: quadratic mean diameter; N: number of trees per ha; Min: minimum diameter; Max: maximum diameter; SD: standard deviation

Finite mixture model (FMM)

Assume a finite mixture model (FMM) comprising of k-components; then the distribution of the *j*th individual component in the mixture is given by a specific probability density function (pdf), $f_j(x)$, and the overall pdf, $f(x)$, for the mixture model can be mathematically represented as:

$$f(x,\pi) = \sum_{j=1}^{k} \pi_i f_j(x) = \pi_1 f_1(x) + \pi_2 f_2(x) + \cdots + \pi_k f_k(x) \qquad Eq.[1]$$

where $f_j(x)$ is the probability density function (pdf) of the *j*th individual component distribution; *x* is a continuous random variable (i.e., tree diameters) π is the mixing proportion of the components in the mixture for which the condition: $0 \leq \pi_j \leq 1$ and $\pi_k = \sum_{j=1}^{k} \pi_j$, must be satisfied. A more detailed background information on FMM can be found in Liu *et al.* (2002). Finite mixture model provides better estimation of diameter distribution that is multimodal or heavily skewed than single component univariate models. In addition, FMM is also useful for classification purpose.

Five finite mixture models comprise of gamma, Gompertz, log-Logistic, lognormal and Weibull were evaluated in this study. The component pdfs in the finite mixture models are the two-parameter gamma, two-parameter Gompertz, two-parameter log-logistic, two-parameter lognormal and two-parameter Weibull functions. The number of k-component in the mixture can be iteratively searched (ZHANG *et al*., 2001; OGANA, 2018) or predetermined (ZASADA; CIESZEWSKI, 2005; LIU *et al.*, 2014). In this study, iterative search was used to determine the optimum number of components in the mixture. In the iterative search, different number components ranging from 1 to 5 were evaluated based on minimum Bayesian Information Criterion (BIC). The number of components with the minimum BIC is selected. Two components in each mixture was adequate for the *G. arborea* stands. The pdfs of the models are expressed as:

Gamma mixture:

$$f(x) = \pi_1\left(\frac{x^{\alpha_1-1}}{{\beta_1}^{\alpha_1}\Gamma(\alpha_1)}exp\left(-\frac{x}{\beta_1}\right)\right) + \pi_2\left(\frac{x^{\alpha_2-1}}{{\beta_2}^{\alpha_2}\Gamma(\alpha_2)}exp\left(-\frac{x}{\beta_2}\right)\right) \qquad \text{Eq. [2]}$$

Gompertz mixture:

$$f(x) = \pi_1\left(\beta_1 exp(\alpha_1 x)exp\left(\frac{\beta_1 exp(\alpha_1 x)-1}{\alpha_1}\right)\right) + \pi_2\left(\beta_2 exp(\alpha_2 x)exp\left(\frac{\beta_2 exp(\alpha_2 x)-1}{\alpha_2}\right)\right) \qquad \text{Eq. [3]}$$

Log-Logistic (*LogL*) *mixture*:

$$f(x) = \pi_1\left(\frac{\alpha_1}{\beta_1}\left(\frac{x}{\beta_1}\right)^{\alpha_1-1}\left[1+\left(\frac{x}{\beta_1}\right)^{\alpha_1}\right]^{-2}\right) + \pi_2\left(\frac{\alpha_2}{\beta_2}\left(\frac{x}{\beta_2}\right)^{\alpha_2-1}\left[1+\left(\frac{x}{\beta_2}\right)^{\alpha_2}\right]^{-2}\right) \qquad \text{Eq. [4]}$$

Lognormal mixture:

$$f(x) = \pi_1\left(\frac{\exp\left(-\frac{1}{2}\left(\frac{lnx-\alpha_1}{\beta_1}\right)^2\right)}{x\beta_1\sqrt{2\pi}}\right) + \pi_2\left(\frac{\exp\left(-\frac{1}{2}\left(\frac{lnx-\alpha_2}{\beta_2}\right)^2\right)}{x\beta_2\sqrt{2\pi}}\right) \qquad \text{Eq. [5]}$$

Weibull mixture:

$$f(x) = \pi_1\left(\frac{\alpha_1}{\beta_1}\left(\frac{x}{\beta_1}\right)^{\alpha_1-1}\exp\left[-\left(\frac{x}{\beta_1}\right)^{\alpha_1}\right]\right) + \pi_2\left(\frac{\alpha_2}{\beta_2}\left(\frac{x}{\beta_2}\right)^{\alpha_2-1}\exp\left[-\left(\frac{x}{\beta_2}\right)^{\alpha_2}\right]\right) \qquad \text{Eq. [6]}$$

where α_1, β_1 are the parameters of the distribution of the first component in the mixture; α_2, β_2 are the parameters of the distribution of the second component in the mixture; π is the mixing proportion; the numeric subscript 1, 2 represent the first and second components in the mixture, respectively, *x* is the random variable (Dbh), and $\Gamma(\bullet)$ is the gamma function. The method of maximum likelihood by the expectation maximization algorithm was used to fit the FMMs to the four *G. arborea* stands. The analysis was carried with the 'ForestFit' package (TEIMOURI, 2020) implemented in R (R CORE TEAM, 2017).

Method of evaluation

Five goodness-of-fit indices were used to evaluate the finite mixture models. For each model, the Akaike Information Criterion (AIC), Hannan-Quinn Criterion (HQC), Anderson-Darling (AD), Cramer-von Mises (W^2) and Kolmogorov-Smirnov (KS) statistics were calculated. The smaller the statistics are, better the FMM.

Akaike Information Criterion (AIC):

$$AIC = -2LL + 2p \qquad \text{Eq. [7]}$$

Hannan-Quinn Criterion (HQC):

$$HQC = -2LL + plnln(n) \qquad \text{Eq. [8]}$$

Kolmogorov-Smirnov (KS) statistics:

$$KS = max\{max_{1\le i\le n_i}[F_n(x_i) - F_0(x_j)], max_{1\le i\le n_i}[F_o(x_j) - F_n(x_{i-1})]\} \qquad \text{Eq. [9]}$$

Anderson-Darling (AD) statistic:

$$AD = -n_i - \sum_{j=1}^{n}(2j-1)\left[ln\left(F_o(x_j)\right) + ln(1 - F_n(x_{i-1}))\right]/n_i \qquad \text{Eq. [10]}$$

Cramer-von Mises (W^2) statistic:

$$W^2 = \sum_{i=1}^{n}\left\{\hat{F}(x_i) - \frac{(i-0.5)}{n}\right\}^2 + \frac{1}{12n} \qquad \text{Eq. [11]}$$

where $F(x_i)$ represents the observed cumulative frequency distribution; x_i the diameter (in cm, i ranged from 1 to n); i represents the individual; n the number of observation; $F_0(x_i)$ is the theoretical cumulative frequency distribution; LL is the log-likelihood value returned after optimization; *ln* is the natural logarithm.

Ranking of the FMM

This study utilised the relative rank introduced by Poudel and Cao (2013) and is expressed as:

$$R_i = 1 + \frac{(m-1)(S_i - S_{min})}{S_{max} - S_{min}} \qquad \text{Eq. [12]}$$

where R_i is the relative rank of model i (i = 1, 2, …, m); m is the number of model assessed (5 FMMs), S_i is the fit index value of model i; S_{max} and S_{min}, respectively are the maximum and minimum values of S_i. Relative rank is a real number between 1 (best) and 5 (worst). For each FMM, the relative ranks were totalled for the five goodness-of-fit indices. This was then analysed using One-way analysis of variance (ANOVA) at 5% level of significant. Fisher least significant difference (Fisher-LSD) was used to separate models that were significantly different.

The best finite mixture models based the on the relative rank sum were used in conjunction with appropriate height-diameter model and the volume equation developed for the *G. arborea* stand by Ogana and Ekpa (2020) to estimate the stand density and volume by class for the stands.

Nalund HD: $H = \frac{D^2}{(2.850715 + 0.100139D)^2}$; MAB = 3.075; RMSE = 3.944 Eq. [13]

Volume equation: $V = 1.534\, x\, 10^{-5} D^{2.162} H^{1.244}$; RMSE = 0.0842; MAB = 0.0019 Eq. [14]

where H = average tree height (m); D = class diameter (cm); V = volume (m^3)

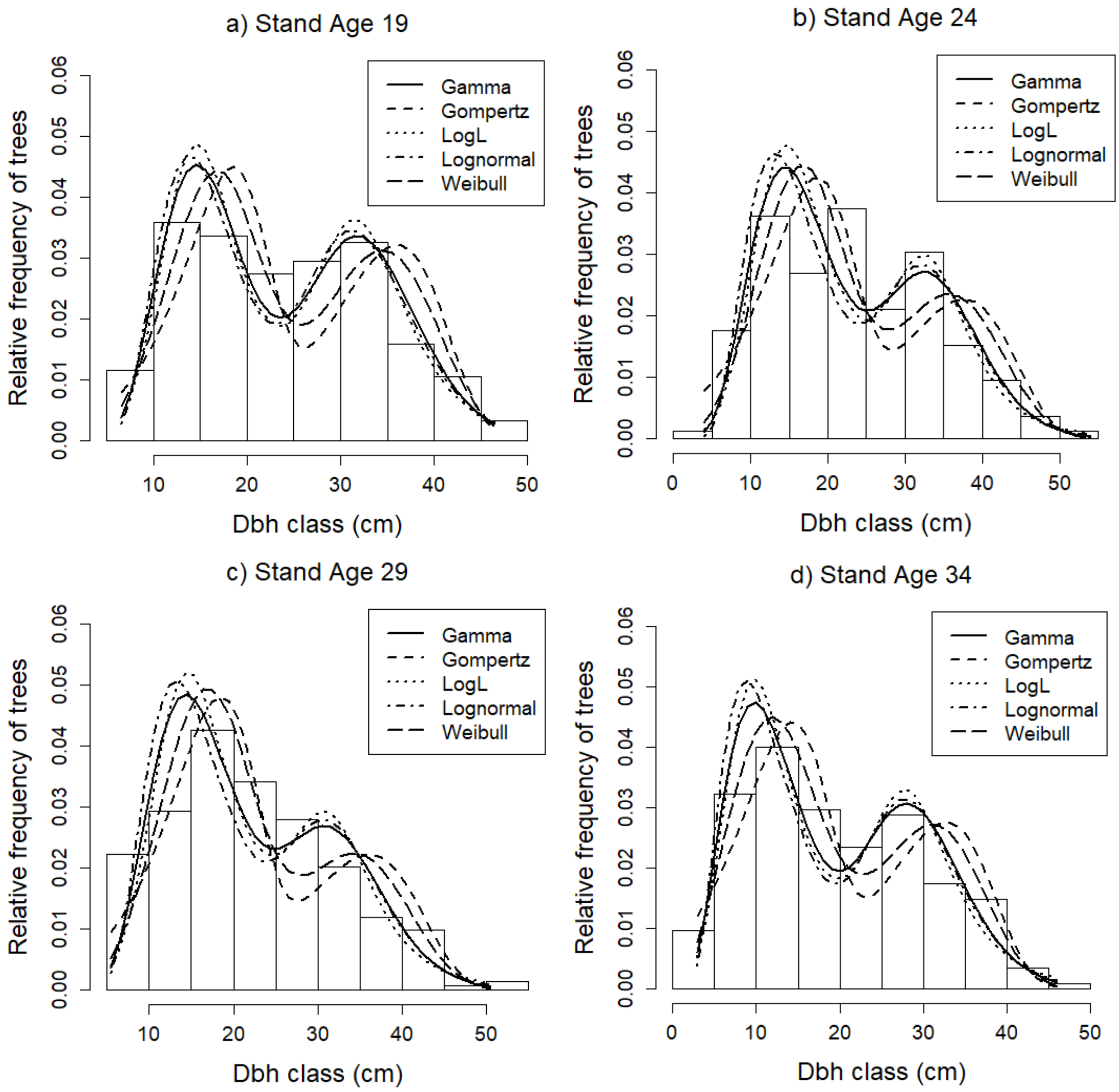
RESULTS

The parameters including the mixing proportion (π) of the five FMM are presented in Table 2. For all models, the estimated mixing proportion satisfied the condition stated in the methodology i.e., summed up to 1. The gamma mixture had larger π value in the first component for the youngest stand (age 19) compared to the older stand. The π values from Gompertz mixture were larger only in the 1st component in stand age 19 and 24. Lower values were observed in the other stand compared to the 2nd component. This value was relatively low in the 1st component of the LogL mixture compared to the 2nd component in most of the stands. Contrarily, in lognormal mixture, the π values were larger in the 1st component compared to the 2nd component in three of the stands. The Weibull mix had lower π in the 1st component in stand age 19 and 24, and larger in stand age 24 and 34 compared to the second components. The estimates of other parameters of the models were also reasonable.

Table 2: Estimated parameters of the finite mixture models in the four stands
Tabela 2: Parâmetros estimados dos modelos de mistura finita nos quatro povoamentos

Stands	**FMM**	**1st component**			**2nd component**		
		π_1	α_1	β_1	π_2	α_2	β_2
Age 19	Gamma	0.5325	10.8410	1.4792	0.4675	32.9815	1.0021
Age 24		0.3763	31.9443	1.0756	0.6237	7.6978	2.1512
Age 29		0.3498	31.5992	1.0385	0.6502	8.3723	1.9512
Age 34		0.4228	25.3962	1.1653	0.5772	5.2452	2.3069
Age 19	Gompertz	0.5195	0.1678	0.0004	0.4805	0.2274	0.0036
Age 24		0.4505	0.1357	0.0008	0.5495	0.1827	0.0065
Age 29		0.5744	0.1981	0.0056	0.4256	0.1420	0.0009
Age 34		0.4676	0.1577	0.0009	0.5324	0.1920	0.0128
Age 19	LogL	0.4567	9.6918	32.4656	0.5433	5.4311	15.6483
Age 24		0.6428	4.5716	16.1406	0.3572	9.8598	33.5322
Age 29		0.3304	9.6526	31.9961	0.6696	4.7651	16.0208
Age 34		0.3985	8.6603	29.1450	0.6015	3.6557	11.5748
Age 19	Lognormal	0.5414	2.7280	0.3170	0.4586	3.4829	0.1740
Age 24		0.3728	3.5212	0.1753	0.6272	2.7209	0.3813
Age 29		0.6641	2.7325	0.3688	0.3359	3.4750	0.1761
Age 34		0.6012	2.4047	0.4743	0.3988	3.3778	0.1925
Age 19	Weibull	0.4990	5.9546	35.5466	0.5010	3.9446	17.7471
Age 24		0.5712	3.5171	18.0508	0.4288	5.4229	37.0408
Age 29		0.3993	5.4064	35.3921	0.6007	3.7140	17.9409
Age 34		0.5592	2.7303	13.7850	0.4408	5.3137	32.3187

The relative frequency of trees against diameter classes of the observed diameter distributions and the fitted FMMs in the stands are shown in Figure 1a to d. Bimodalities were obvious in stand age 19, 24 and 34 that is, Figure 1a, 1b and 1d. While bimodal structure was not obvious in stand age 29 (Figure 1c). In all cases, the FMMs predicted well in most of the diameter classes, especially the larger classes. They however, overestimated the relative frequency of trees in diameter class of 15 cm.



a) Stand Age 19

b) Stand Age 24

c) Stand Age 29

d) Stand Age 34

Figure 1. Observed and fitted finite mixture models in the four stands.
Figura 1. Modelos de mistura finite observados e ajustados nos quatro povoamentos.

The evaluation of the quality of fits produced by the FMMs are presented in Table 3. The results showed the gamma mixture had the smallest indices and relative ranks (values in parenthesis, 1.00) in stand age 19, 24 and 34. Whereas Gompertz mixture had the largest values of the indices and relative ranks (5.00) in those stands. The Weibull mixture was slightly better than the gamma in stand age 29; while the lognormal mixture had the worst fit in the stand.

Table 3. Evaluation statistics and the relative rank of the finite mixture models in the four stands
Tabela 3. Estatísticas de avaliação e classificação relativa dos modelos de mistura finite nos quatro povoamentos

FMM	**Stand Age 19**					**Stand Age 24**				
	AIC	**HQC**	**AD**	**W^2**	**KS**	**AIC**	**HQC**	**AD**	**W^2**	**KS**
Gamma	1402	1408	0.3901	0.0611	0.0507	1289	1296	0.5521	0.1012	0.0741
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(2.97)
Gompertz	1422	1429	1.3813	0.2037	0.0786	1306	1312	1.3433	0.1957	0.0814
	(5.00)	(5.00)	(5.00)	(5.00)	(5.00)	(5.00)	(3.78)	(5.00)	(4.85)	(4.03)
LogL	1408	1415	0.5071	0.0820	0.0560	1292	1298	0.5940	0.1088	0.0777
	(2.20)	(2.33)	(1.47)	(1.59)	(1.75)	(1.71)	(1.35)	(1.21)	(1.31)	(3.49)
Lognormal	1405	1411	0.5362	0.0910	0.0614	1296	1303	1.0391	0.1993	0.0882
	(1.60)	(1.57)	(1.59)	(1.84)	(2.53)	(2.65)	(2.22)	(3.46)	(5.00)	(4.99)
Weibull	1405	1411	0.6911	0.0975	0.0531	1293	1299	0.8699	0.1448	0.0604
	(1.60)	(1.57)	(2.21)	(2.02)	(1.35)	(1.94)	(1.52)	(2.61)	(2.78)	(1.00)
	Stand Age 29					**Stand Age 34**				
Gamma	2120	2127	1.6442	0.2956	0.0885	1701	1708	0.9039	0.1439	0.0564
	(1.00)	(1.00)	(1.94)	(2.54)	(3.52)	(1.00)	(1.00)	(1.37)	(1.57)	(1.01)
Gompertz	2141	2149	1.7587	0.2352	0.0673	1722	1729	1.3974	0.2160	0.0839
	(5.00)	(5.00)	(2.36)	(1.72)	(1.86)	(5.00)	(5.00)	(4.56)	(3.62)	(5.00)
LogL	2128	2136	1.8340	0.2976	0.0864	1712	1719	1.2164	0.1827	0.0604
	(2.52)	(2.64)	(2.64)	(2.57)	(3.35)	(3.10)	(3.10)	(3.39)	(2.67)	(1.58)
Lognormal	2128	2135	2.4729	0.4759	0.1074	1707	1714	1.4646	0.2646	0.0786
	(2.52)	(2.45)	(5.00)	(5.00)	(5.00)	(2.14)	(2.14)	(5.00)	(5.00)	(4.23)
Weibull	2122	2129	1.3900	0.1824	0.0563	1703	1710	0.8474	0.1238	0.0644
	(1.38)	(1.36)	(1.00)	(1.00)	(1.00)	(1.38)	(1.38)	(1.00)	(1.00)	(2.16)

Values in parenthesis are relative rank

The bar graph of the relative rank sums (mean ± standard errors) of gamma, Gompertz, LogL, lognormal and Weibull mixtures are shown in Figure 2. Gompertz mixture had the largest relative rank sum. This was followed by lognormal, LogL and Weibull mixtures. The gamma mixture had the smallest relative rank sum. The confidence intervals were wider in lognormal and Gompertz mixtures. The result from the ANOVA showed a significant difference ($p < 0.05$) in the relative rank sums of the five FMMs. The gamma, Weibull and lognormal mixtures were not significant. The gamma and Weibull were significantly different from lognormal. But LogL and lognormal were not significant. All FMMs were significantly different from the Gompertz mixture. Thus, the gamma mixture had the overall best performance while Gompertz had the worst fit. The order of ranking of the FMMs can be summarised as: gamma followed by Weibull, LogL, lognormal and Gompertz.

The three best FMMs (i.e., gamma, Weibull and LogL) were used to quantify the overall density and volume of the *G. arborea* stands (see Appendix)

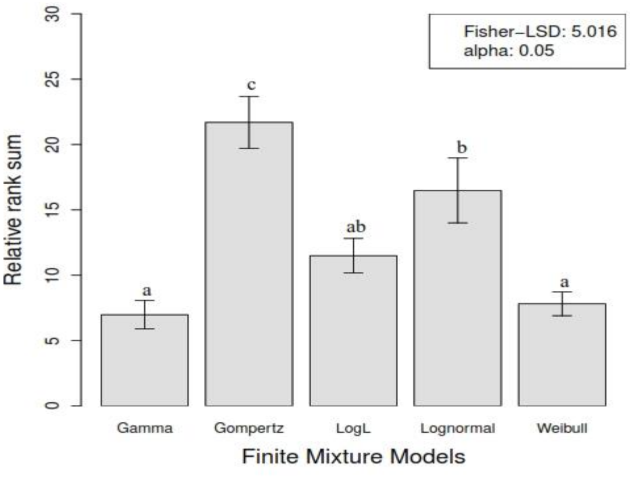


Figure 2. Bar graph of relative rank sum of the finite mixture models. Models with the same letter are not significantly different.

Figura 2. Gráfico de barras da soma relativa da classificação dos modelos de mistura finita. Modelos com a mesma letra não são significativamente diferentes.

DISCUSSION

Anthropogenic and natural factors such as bush burning, illegal exploitation, wind, and the like are good examples of forest disturbances (TSOGT; LIN, 2012) that affect non-spatial structure of forest stands. Podlaski (2017) asserted that forest disturbances create gap in the forest which can result to heavily skewed or multimodal (more than one mode) diameter distributions. This is evidence in the bimodal structures of stand age 19, 24 and 34 of the *G. arborea* plantations. Bush burning and illegal/uncontrolled exploitation are the main anthropogenic factors affecting most of the forest stands in Nigeria including the *G. arborea* stands (AJAYI, 2013; EKPA *et al*., 2014; OGANA, 2018).

The bimodal structures in the stands were well represented by the two components of the finite mixture models especially the larger diameter classes, which implies good valuation of the harvested wood. The estimate of the mixing proportion (π) indicates the contribution of the individual component to the overall diameter distribution of the stands (OGANA, 2018). The estimate π varies among the five models which also reflected in their performances with gamma mixture having the best fits. Only in stand age 29 that the Weibull mixture performed slightly better than the gamma mixture. The structure of stand age 29 seems to be more of a unimodal (single peak) than bimodal structure. The Weibull distribution approximate unimodal diameter distribution better than gamma (MATAJI *et al*., 1999; OGANA *et al*., 2015; MIRZAE *et al*., 2016). However, better results were observed for the gamma mixture in stand with obvious bimodal structures. Zasada and Cieszewski (2005) found gamma mixture to be superior to normal and lognormal for characterising diameter distribution by tree social class in Scots pine. They utilised a natural classification scheme for which a tree could either belong to the dominant class (dominant and codominant trees) or dominated class (intermediate and suppressed trees). However, in this study, iterative search was used to determine the number of components required for better description of the overall diameter distributions of the *G. arborea* stands. Furthermore, Jaworski and Podlaski (2012) reported that gamma and Weibull mixtures were equally appropriate for describing the irregular and multimodal diameter distributions of forest stands. Similarly, Ogana (2018) also reported good fits with gamma and Weibull mixtures in the degraded *G. arborea* stands of Omo Forest Reserve. The study at hand also shows no significant difference in the relative rank sum of the overall fits of gamma and Weibull mixtures.

Of the FMMs evaluated in this study, application of the Gompertz and log-logistic mixtures have been relative few especially in forestry literature. The relative rank sum of the log-logistic was not significantly different from the gamma and Weibull mixtures. This implies that the log-logistic mixture could be used in lieu of those mixtures to describe the overall diameter distributions of the disturbed *G. arborea* stands. Just as with the Weibull, the log-logistic has a closed form cumulative distribution function; as such, does not require numerical integration to estimate the relative frequencies of trees in diameter class. The performance of Gompertz mixture was relatively poor and is not suitable for the forest stands.

CONCLUSION

- The bimodal diameter distributions of the disturbed *G. arborea* stands have been described with two components gamma, Gompertz, log-logistic, lognormal and Weibull mixtures.
- Of the mixtures evaluated, gamma, Weibull and log-logistic are the recommended models for the *G. arborea* stands in Oluwa Forest Reserve. The Gompertz and lognormal mixtures are unsuitable for the stand.
- Application of the best finite mixture models provided good prediction of the forest stand volume.

REFERENCES

ADEKUNLE, V. A. J. Conservation of tree species diversity in tropical rainforest ecosystem of south-west Nigeria. **Journal of Tropical Forest Science**, Kuala Lumpur, vol. 18, n. 2, p. 91-101, Nov. 2006.

AJAYI, S. Diameter Distribution for *Gmelina arborea* (Roxb) plantation in Ukpon River Forest Reserve, Cross River state, Nigeria. **AFREV STECH (An International Journal of Science and Technology),** Addis Ababa, vol. 2, n. 1, p. 64-82, Jan. 2013.

EKPA, N. E.; AKINDELE, S. O.; UDOFIA, S. I. Gmelina arborea Roxb. graded stands with the Weibull distribution Function in Oluwa Forest Reserve, Nigeria. **International Journal of Agroforestry and Silviculture**, Islamabad**,** vol. 1, n. 9, p. 110-113, Sep. 2014.

JAWORSKI, A.; PODLASKI, R. Modelling irregular and multimodal tree diameter distributions by finite mixture models: an approach to stand structure characterisation. **Journal of Forest Research**, London, vol. 17, n. 1, p. 79-88, Mar. 2012.

LIU, F.; LI, F.; ZHANG, L.; JIN, X. Modelling diameter distributions of mixed-species forest stands. **Scandinavian Journal of Forest Research**, Stockholm, vol. 29, n. 7, p. 653-663, Oct. 2014.

LIU, C.; ZHANG, L.; DAVIS, C. J.; SOLOMON, D. S.; GOVE, J. H. A finite mixture model for characterizing the diameter distribution of mixed-species forest stands. **Forest Science**, Washington, vol. 48, n. 4, p. 653-661, Nov. 2002.

MATAJI, A.; HOJJATI, M.; NAMIRANIAN, M. Study on number distribution in diameter classes in natural forest by using probability distribution. **Iran Journal of Natural Resources**, Tehran, vol. 53, n. 2, p. 165-171, 1999.

MIRZAEI, M.; AZIZ, J.; MAHDAVI, A.; RAD, A. M. Modelling frequency distributions of tree diameter, height and crown area by six probability functions for open forests of *Quercus persica* in Iran. **Journal of Forestry Research**, Harbin, vol. 27, n. 4, p. 1-6, Aug. 2016.

OGANA, F. N. Application of finite mixture to characterise degraded *Gmelina arborea* Roxb plantation in Omo Forest Reserve, Nigeria. **Journal of Forest and Environmental Science**, Seoul, vol. 34, n.6, p. 451-456, Dec. 2018.

OGANA, F. N.; EKPA, N. E. Modelling the non-spatial structure of Gmelina arborea Roxb stands in Oluwa Forest Reserve, Nigeria. **Forestist**, Istanbul, vol. 70, n.2, p. 133-140, Apr. 2020.

OGANA, F. N.; CHUKWU, O.; AJAYI, S. Tree size distribution modelling: moving from complexity to finite mixture. **Journal of Forest and Environmental Science**, Seoul, vol. 36, n.1, p. 7-16, Mar. 2020.

OGANA, F. N.; ITAM, E. S.; OSHO, J. S. A. Modelling diameter distributions of *Gmelina arborea* plantation in Omo Forest Reserve, Nigeria with Johnson's S_B. **Journal of Sustainable Forestry**, London, vol. 36, n. 2, p. 123-133, Mar. 2017.

OGANA, F. N.; OSHO, J. S. A.; GORGOSO-VARELA, J. J. Comparison of beta, gamma and Weibull distributions for characterising tree diameter in Oluwa Forest Reserve, Ondo State, Nigeria. **Journal of Natural Sciences Research**, Islamabad, vol. 5, n. 4, p. 28-36, Mar. 2015.

ONYEKWELU, J. C.; MOSANDL, R.; STIMM, B. Productivity, site evaluation and state of nutrition of *Gmelina arborea* plantation in tropical rainforest zone in south-western Nigeria. **Forest Ecology and Management**, Amsterdam, vol. 229, n. 1-3, p. 214-227, Jul. 2006.

PODLASKI, R. Forest modelling: the gamma shape mixture model and simulation of tree diameter distributions. **Annals of Forest Science**, Paris, vol. 74, n. 2, p. 29, Apr. 2017.

POUDEL, K. P.; CAO, Q. V. Evaluation of methods to predict Weibull parameters for characterising diameter distributions. **Forest Science**, Washington, vol. 59, n. 2, p. 243-252, Apr. 2013.

R CORE TEAM **R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria**. Available at: <http://www.R-project.org/>. Accessed on: 30/06/2017

TEIMOURI, M. **ForestFit: Statistical modelling for plant size distributions. R package version 0.5.7**. Available at: <https://CRAN.R-project.org/package=ForestFit> Accessed on: 15/03/2020.

TSOGT, K.; LIN, C. A flexible modelling of irregular diameter structure for the volume estimation of forest stands. **Journal of Forest Research**, London, vol. 19. n, 1, p. 1-11, Nov. 2014.

ZASADA, M.; CIESZEWSKI, C. J. A finite mixture distribution approach for characterising tree diameter distributions by natural social class in pure even-aged Scots pine stand in Poland. **Forest Ecology and Management**, Amsterdam, vol. 204, n. 2-3, p. 145-158, Jan. 2005.

ZHANG, L.; LIU, C. Fitting irregular diameter distributions of forest stands by Weibull, modified Weibull, and mixture Weibull models. **Journal of Forest Research**, London, vol. 11, p. n. 5, 369-372, Oct. 2006.

ZHANG, L.; GOVE, J. H.; LIU, C.; LEAK, W. B. A finite mixture of two Weibull distributions for modelling the diameter distributions of rotated-sigmoid, uneven-aged stands. **Canadian Journal of Forest Research**, Ottawa, vol. 31, n. 9, p. 1654-1659, Sep. 2001.