SIXTH-TREE SAMPLING IN FOREST INVENTORIES

Dieter R. Pelz*

SUMMARY

Tree distance samling methods have been used for several years in forest inventories, for estimating the number of trees or the basal area. In practical applications the methods are easy to use, all variables of interest can be measured. In calculating the average basal area or number of trees a ratio estimator has to be used.

The accuracy of these methods depends on the distribution of the trees over the area. Especially in clumped populations a bias can occur that cannot be controlled. Although some methods exist that can partially correct this bias, no general applicable method has been found yet. The amount of bias cannot be estimated, i.e. it is very difficult to evaluate when these methods give satisfactory results and when not.

1. INTRODUCTION

The distribution of individuals over an area is of interest in many different fields. The distribution of grass-seeds, corn stalks on a field, animals over an area, or trees in a forest has been examined in the past. A large volume of literature exists in ecology, statistics and forestry on distance sampling, giving a number of different estimators for the density and the variance under specific assumptions.

This paper will concentrate on forestry applications, and discuss first the estimation of the number of trees and then the estimation of the basal area with distance measures.

2. ESTIMATION OF STAND DENSITY

First attempts to estimate the number of trees in a forest were made as far bach as 1860 by PRESSLER and 1864 by KOENIG (as cited by PRODAN, 1965). The derived a distance measure by the relation of diameter(d) and circunference(s) of a tree to the distance between trees, and constructed quadrats proportional to d resp. s and estimated the number of trees by

$N = \frac{1}{a^2}$ where a^2 is the area occupied by 1 tree.

Other authors followed a somewhat different approach, they did not consider the size of the trees in defining the area available to one tree.

In general the following two approaches can be applied:

1. measurement from a random point to a tree (Fig. 1)
2. measurement from a center tree to a neighboring tree (Fig. 2)

2.1. Traditional Approaches

As cited by Prodan (196),
Bauernsachs (1942) defined the area available to one tree as:

$$a^2 = \frac{10.000}{N}$$

$$a = \sqrt{\frac{10.000}{N}} \quad \text{or} \quad N = \frac{10.000}{a^2}$$

a^2 is then the average area for one tree. With empirical studies he attempted to find the distance to a tree that closely correlated with the a from above. He found the distance to the 2nd closest tree most appropriate, by using a correction factor of 0.85 he obtained his "true" average distance.

KÖHLER (1952) found that the second tree-tree distance always overestimated the average distance and used the average between the 2. and 3. closest

* Professor of Forest Biometry, University of Freiburg, Germany.

tree, and derived the factor k = 0.8464. WECK (1955) derived another correction factor, his results being based on different stands. STOFFELS (1955) introduced the following relationship that appears intuitively obvious:

Using a random point as center one can define semple plots with the radius to the 1sd 2nd 3rd .. neighbor each containing 0,5, 1,5, 2,5 etc. trees.

The formula for calculating the number of trees then is

for random pt-tree for tree to tree

$$\frac{10^4}{\pi a_1^2} \cdot 0{,}5 \qquad \frac{10^4}{\pi a_1^2} \cdot 1{,}5$$

$$\frac{10^4}{\pi a_2^2} \cdot 1{,}5 \qquad \frac{10^4}{\pi a^2} \cdot 2{,}5$$

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$$\frac{10^4}{\pi a_i^2} (i—0{,}5) \qquad \frac{10^4}{\pi a_i^2} (i+0{,}5)$$

The relation between these two estimates has been described by Pelz (1968).

Based on these basic relatioships a number of inventory methods have been developed, the most important being the 6-th tree method and the method for estimating the stand density by HAUSBURG (1965). HAUSBURG studies the distances of order 1-8 and found that they are correlated with each other. He suggested that the third distance from a tree to a tree performed well and that this could be used for an estimate of the number of trees/ha.

2.2. Bias of tree distance estimates

Before proceeding to the estimation of basal area a few problems, on a practical and theoretical level with these methods shall be mentioned.

As shown by many empirical studies and actual applications in forest inventories these methods work quite satisfactory under a wide variety of conditions. However, theoretical derivations show that these methods can give biased results, under specific assumptions about the underlying distribution of the trees over the area and about independence of the distances and diameter or basal area of the trees. One distribution that is most often assumed is the POISSON distribution. If we define small areas, say 1 m^2 size, and observe whether a tree is on these areas or not then the number of trees follows a POISSON distribution. Under this assumption it can be shown that the correct estimate for the number of trees is

$$N = \frac{10^4}{\pi a_n^2} \cdot n \quad \text{where} \quad n = \begin{cases} \text{distance} \\ \text{order} \end{cases}$$

This would indicate that the formula for estimating the number of trees given by STOFFELS has a bias of bout 10%. This can be accepted as theoretical value, however trees rarely follow a POISSON distribution. SMALTSCHINSKI (1982) studies a stand remeasured several times between 1906 and 1963 and found that only at the end when the stand began to disolve a distribution approximating POISSON could be found, before that other distributions applied. Starting from the observation that the arithmetic mean of the distances squared always over estimated and the harmonic mean always underestimated the true density, Smaltschinski (1982) suggests an estimatation formula for the number of trees:

$$N = \frac{n\,\bar{r}_n^{-2} + (n\text{-}1)\,\overline{\left(\frac{1}{r_n^2}\right)}^{-1}}{\pi\left(\bar{r}_n^{-4} + \overline{\left(\frac{1}{r_n^2}\right)}^{-2}\right)} \cdot 10^4$$

where

N = number of trees

$\bar{r}^{-2}$ = arithm. mean of distance squared

$\frac{1}{\overline{r^2_n}}$ = harmonic mean of distance squared

n = distance order

A test of this estimator for the number of trees showed that it performs well over a variety of stand structures from a Pielou Index of 0,6 to 0,8 (1,0 would mean completely random). There are some uncertainties with distance methods as shown by the previous discussion. On a more intuitive level consider a clumped population where the clumps consist of a least 5 trees. If we now use the 3 rd. distance for tree-tree measure we would overestimate the density because we only measure inside the clumps. Similarly, if we use point-tree methods we can get biased results if we never measure beyond the clumps. There is still research necessary to define exactly for what underlying distributions these method give unbiased results, and what estimators should be used for specific distributions.

3. ESTIMATION OF BASAL AREA

If we now consider the estimation of basal area we get an additional point to consider, namely the correlation between basal area and the distance to the i^{th} tree. Studies have shown that they are correlated.*

3.1. 6-th tree method

Let us now specifically discuss the 6-th tree method. It was developed in the late sixties in Freiburg and firstly described by PRODAN (1969).

The method measures the distance from a randomly chosen point to the sixth closest tree. The basal area is calculated as:

$$B = \frac{\frac{\pi}{4}\left(d^2_1 + d^2_2 + \frac{1}{2} d^2_6\right.}{\pi r^2_6} \cdot 10^4$$

$$= \frac{d^2_1 + d^2_2 + \ldots \frac{1}{2} d^2_6}{r^2_6} \; 2500$$

If we now consider more than 1 point we have to estimate the average basal area by means of the ratio estimate, as we are dealing with the ratio of the tree basal areas to the area these 5,5 tree occupy:

the average area is calculated as

$$\bar{B} = \frac{\Sigma b_i}{\Sigma r^2_i} \cdot \frac{10^4}{4} = \frac{\Sigma b_i}{\Sigma r^2_i} \cdot 25000$$

where

$$b_i = d^2_{1,i} + d^2_{2,i} + d^2_{3,i} + d^2_{4,i} + d^2_{5,i} + d^2_{6,i}$$

and

r^2_i = the distance to the sixth neigbor squared

The variance of the ratio estimate ca nthen be calculated:

$$S^2 = \frac{\bar{r}^{-2}}{n^{-1}} \left(\frac{\Sigma b^2_i}{b^{-2}} + \frac{\Sigma a^2_i}{b^{-2}} - \frac{2\Sigma a_i b_i}{ab}\right)$$

where

$$r = \frac{\Sigma b_i}{\Sigma r^2_6} = \frac{\Sigma b_i}{\Sigma a_i} \quad a_i = r^2_6$$

$$S^2_{\bar{r}} = \frac{S^2}{n}$$

and finally the variance is:

$$S^2_B = a^2 S^2_{\bar{r}} + \bar{r}^{-2} S^2_{a_1}$$

* Unpublished report Dr. Ko, Abteilung für Forstl. Biometrie Freiburg, Germany.

This some what complicated procedure has to be followed if we have an ratio estimate. Presently we are in the process of testing the relation a_i and b_i, we found them related by a straight line for the stands examined, but it does not always pass through te origin, as required for the ratio estimator. If a regression estimator is used the variance s^2 can be calculated by a different formula.

With this method other variables can be included also, such as number of trees, volume, quality etc.

The main advantage of using this method in inventory is the ease of fieldwork, only the diameters of 6 trees and the distance to the 6-th tree have to be measured, there is no need to worry about plot boundary, whether trees are in or out and similar things. Experience has shown in the stand inventories in southern Germany that also untrained field personel can do the inventory work fast and accurate In applying the method one has to realize that we deal with very small plots (often 1/5 to 1/10 of a 1/10 ha plot that is being used frequently as a result the variance will be very high, we often have coefficients of variation that are more than 100%.

4. RESUMO

Métodos de amostragem baseados em distâncias entre árvores tem sido usados já há algum tempo em inventários florestais, para se estimar o número de árvores ou a área basal. Na prática estes métodos são de fácil aplicação e todas as variáveis de interesse podem ser medidas. Para se calcular a área basal média ou número de árvores, uma razão como estimador tem que ser usada. A precisão destes métodos depende da distribuição espacial das árvores sobre a área. Especial em populações onde ocorrem agrupamentos de árvores podem ocorrer tendências nos estimadores, que não podem ser controladas. Apesar de alguns métodos já existirem, que podem corrigir parcialmente esta tendência, não foi ainda desenvolvido um método geral para resolver o problema. O quanto de tendência não pode ser estimado ou seja, é muito difícil avaliar quando estes métodos dão resultados satisfatórios ou não.

5. LITERATURE

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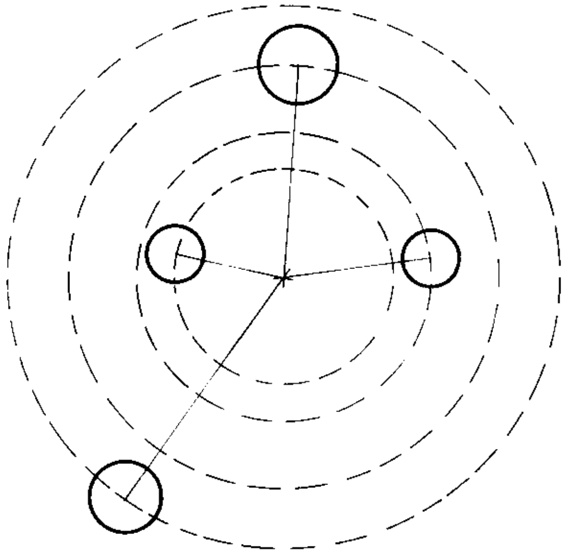

Figura 1: Point-tree distance method.

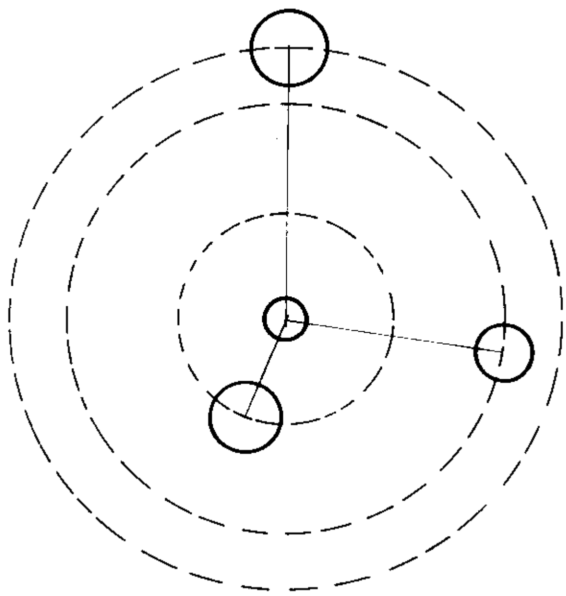

Figura 2: Tree-tree distance method.