THREE-P-SAMPLING FOR FOREST INVENTORY.

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SUMMARY

As 3-P sampling works with individual sample trees, one has to visit and estimate all trees in the population and then select the ones that should be measured exactly. The unegual selection probability results in a higher efficiency than with other methods, say simple random sampling, but the method certainly will not be suitable to all forest conditions. Although 3-P has been developed for timber sales inventories many other applications have been reported in the literature:

In extensive forest inventories two stage sampling is being used, the first stage selects plots or points (in point sampling) and in the second stage the sample trees on these plots are being selected by 3 P.

Other applications include sampling for fuel in forest fire research, sampling for the weight of deer at check deer at checking stations and others. Lund (1975) gives numerous examples of 3-P applications. The method can be applied whenever the variable of interest can be estimated, and then a few individuals measured. Since its introduction by Grosenbaugh in 1963, 3-P sampling proved to be an efficient inventory method that should be considered as a serious alternative, but as with all other sampling method it would be unreasonable to expect it to be applicable under all conditions.

1. INTRODUCTION

Several methods have been developed for the inventory of forest stands, by complete enumeration or by sampling techniques. This paper describes a sampling method that has been employed successfully for a number of years in forest inventories and in timber sales inventories.

The method developed by Grosenbaugh (1963) is called 3 P: Probability Proportional to Prediction and is based on the concept of unequal probability sampling. Let me first say a few words on equal and unequal probability sampling.

2. UNEGAL PROBABILITY SAMPLING

In equal probability sampling each unit has the same probability or chance of being selected. For example, if we want to take a sample of size 40 from a population of size 1000 each unit has the chance of 40/1000 = 0,04 to be included. Assume now that we deal with trees and want to determine the volume of the population. With equal probability sampling each tree has the same chance of being included whether they are very small or very big. It is a well known fact in sampling, and it is intuitivly obvious that it is much better to give larger trees a bigger chance to be included in the sample. The best selection strategy is to use a probability proportional to size, known as PPS sampling.

As example of this selection criterion consider the hypothetical population in Table 1.

Table 1 PPS-sampling with a population o fsize 10

Population:

Tree No.	V in m^3	Cumulative V	Selection range
1	3,02	3,02	0,00 — 3,02
2	4,53	7,55	3,03 — 7,55
3	3,81	11,36	7,56 — 11,36
4	4,05	15,41	11,37 — 15,41
5	3,24	18,65	15,42 — 18,65
6	2,98	21,63	18,66 — 21,63
7	3,29	24,92	21,64 — 24,92
8	3,62	28,54	24,93 — 28,54
9	3,07	31,61	28,55 — 31,61
10	2,83	34,44	31,62 — 34,44

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Let us assume now that we want to select 3 trees with a selection probability proportional to size. We calculate the cumulative volumes to define the range. Assume we select 3 random numbers between 0-34,44 with 24,38-0,016-11,41. The first Number corresponds with tree no. 7, the second with tree no. 1 and the third with tree 4. If we di dthat many times over tree no. 1 would be selected 3,02/34,44 times, the second tree 4,53/34,44 times etc. That means the selection probability is equal to the size of the tree.

This concept that has been shown here with an unrealistic small example has been successfully employed in forest inventories in the form of list sampling as described by Loetsch (1973). There the areas of all forest stands were listed, they were selected for measurement of diameters, heights and volumes with a probability proportional to the area or size of the stand. The method was more efficient than simple random sampling because more larger stands were sampled. The main disadvantage of this method is that the entire population has to be known and listed and the sampling has be done from this list.

3. THREE P-SAMPLING

In some cases the variable "size" can be substituted by another variable. Lahire (1954, as cited by Grosenbaugh 1967) suggested usingan estimation of size instead of size directly. Grosenbaugh extended this idea further and developed a sampling method with a selection probability proportional to the estimation or prediction. He named this method 3 **P** or probability proportional to prediction. Each unit in the population is visited an its size estimated. Based on this estimation one decides whether the unit will be measured exactly or not — the selection probability is proportional to this estimation.

3.1 Three P-Sampling procedures

For the practical implementation of this method the following procedure described for the determination of volume has to be followed.

(1) **Determine the sample size n** the well known formulas for calculating sample sizes can be used. (short version $n = \frac{t^2 s^2}{E^2}$)

(2) **Estimate the total for the population,** in our case the total stand volume. This variable is denoted by $\hat{X}$

(3) **Estimate the maximum,** the max. tree volume in the stand denoted by K

(4) **Calculate the random number range by**

$$Z = \frac{\hat{X}}{n} \quad \text{(we want to select 1 tree for every Z m}^3\text{)}$$

the random number range is then 1 — Z, Steps 1-4 are done prior to the actual sampling.

(5) **sample selection:** Estimate the volume of a tree

(6) Compare this volume X_i with the random number if

$X_i \geqslant RN$ the tree will be included, the volume will be determined exactly

$X_i < RN$ the tree is not included

$X_i \geqslant K$ the tree will be included because he has a probability of selection $= 1$

Repeat for all trees in the population

For calculating the population values we use

$$\hat{Y} = \Sigma X_i \left(\frac{\Sigma \frac{Y_i}{X_i}}{n} \right)$$

The total volume is the sum of all estimations corrected by the average ratio of true: estimated volume.

The standard deviation can be estimated with

$$S_{\hat{y}} = \hat{Y} \sqrt{\frac{\Sigma (R_i - \bar{R}_i)^2}{n-1}} \qquad \text{where} \quad R_i = \frac{Y_i}{X_i}$$

and the estandard error

$$S_{\hat{\bar{y}}} = \frac{Y_{\hat{y}}}{\sqrt{n}}$$

3.2. The following example can illustrate the calculations.

The à priori estimates are

$$k = 17{,}00 \quad \hat{X} = 120$$
$$n = 3$$

The Aandom number range is 1-120/3 = 1-40, that means we want to select 1 sample tree for every 40 m^3 of volume.

Table 2 3-P-Sampling with a population of size 10

Tree No.	Estimated Volume	Random number	Actual Vol.	ratio
1	7,00	12,84	—	
2	16,00	10,55	15,90	0,99375
3	8,00	14,61		
4	10,00	34,28		
5	12,00	5,64	12,57	1,0475
6	14,00	2,34	12,88	0,9200
7	10,00	27,03		
8	7,00	15,34		
9	10,00	8,46	8,24	0,824
10	8,00	18,54		
	102,00			0,9463 125

Population total

$$\hat{Y} = \Sigma X_i \left(\frac{Y_i}{X_i} / n\right)$$

$$= 102 \;.\; 0{,}9463\ 125 = 96{,}5238$$

Standard deviation

$$S_{\hat{Y}} = \hat{Y}_i \; \frac{\Sigma (R_i - \bar{R})^2}{n - 1} = 9{,}3486654$$

Standar error
of total volume

$$S_{\hat{\bar{Y}}} = \frac{9{,}3486654}{\sqrt{4}} = 4{,}67433$$

sampling error CV% = 4,84%

3.3. Precision and Accuracy of 3 P-sampling

With 3-P-sampling the sample size n is a random variable, therefore we can end up with a different sample size as preselected, although the difference in practical applications is in most cases small. In our example we specified n = 3 but obtained n = 4. On a more theoretical level, this is also the reason that the calculated variance is only approximate and not exact, but the approximation is very good (Grosenbaugh, 1976; Schreuder et al. 1971).

The number of measurements needed for an acceptable sampling error of less than 5 % is for many applications less than 100, depending on the variation of the ratio Y_i/X_i. In estimating the volume it is important to be consistent, i. e. to keep the variance of Y_i/Z_i low, wheras biased estimates, e.g. consistenly 10 % too high, do not matter, as this will not influence the variability, and will be corrected by the ratio actual/estimated.

There are several computer samling simulators that allow testing the method and gaining some experience with it (Goldsmith, 1976 — Pelz 1977). In practical applications a short estimation exercise in the beginning normally suffices.

3.4. Sample tree volume estimation

The estimation of volume can be done in one of several ways

3.4.1 Volume estimates

3.4.1.1 Occular estimate — this requires normally some experience, experienced cruisers can estimate quite accurately with little variation.

3.4.1.2 rule of thumb estimate

like $V = D^2/1000$ (formula by Denzin) with some allowance for height. The formula is exact for HF = 12,74, deviations from the standard height of 25-26 m result in subtraction or addition of 3 % per m. (Prodan 1965).

3.4.1.3 local volume table
$V = f(D)$

3.4.1.4 standard volume table or some volume function

3.4.2 Volumes measurements

The true volume of the 3-P sample trees can be determined using

3.4.2.1 standard volume tables
$V = f(D,H)$

but for the particular application a bias may exist which can be considerable, as several studies show

3.4.2.2 Volume equations $V = f(D, H,F)$; $V = f(D_{1,3}, D_o, H)$ with these equations the form of the trees are considered, upper stem diameters used are in practice are O 7 m or D 0,3 h.

3.4.2.3 Sample tree methods

— measure trees with optical dendrometers to determine diameters along the stem.

There are a number of dendrometers available like an optical caliper (Wheeler Pentaprism), Spiegel — Relaskop, Telerelaskop, Breithaupth-Todis (fixed base range finder), BARR and STROUD, and ZEISS-Teletop.

For conditions where labor is relatively expensive dendrometry is the only feasible method, but otherwise the climbing or felling of sample trees could be considered even a better alternative, as with some of the traditional sample tree methods described in the literature.

— Felled Sample trees

For felled sample trees all measurements can be obtained with high accuracy and all well known methods for determining the volume can be used.

The actual volume can be determined from the diameter measurements by one of several methods:

3.4.2.3.1 by sections

the maesured diameters are considered at the middle of a section and the total volume calculated as sum of the volumes of the sections. The accuracy of this method depends on the number of sections and the form o fthe trees.

3.4.2.3. with polynomials of higher order

with this method the form of the tree is approximated and the volume can be obtained by rotating this polynomial, if polynomials of higher order are used one needs several diameter measurements.

3.4.2.3.3 by spline approximation

this is a method where for each section of the tree a polynomial is fitted and these polynomials are connected by a continous function. It is somewhat more flexible than method 2 and gives reasonable results. The method has been described in more detail by Liu (1979) and Hradetzky (1981) — (the computer program for the algorithm can be obtained from our department).

A test of spline approximation in combination with 3-P âielded satisfactory results, we used 4-6 measurement points (including dbh and total height) The number of measurement points is not fixed with this method and it is therefore well suited to optical measurements where the same number of points can not always be clearly seen.

4. RESUMO

Amostragem 3-P trabalha com árvore sindividualmente amostradas. Desta forma torna-se necessário visitar e estimar todas as árvores na população e a partir daí selecionar aquelas que devem ser medidas exatamente. A seleção com diferentes probabilidades resulta numa maior eficiência que em outros métodos, por exemplo em amostragem inteirame-n te aleatória, mas o método certamente não será aplicável a todas condições florestais. Apesar de 3-P ter sido desenvolvido para inventários onde se pretende vender madeira, muitas outras aplicações tem sido apresentadas na literatura: Em inventários extensivos com amostragem em dois estágios, tem sido usada, onde o primeiro estágio seleciona amostras ou pontos (amostragem de Bitterlich) e no segundo estágio as árvores são consideradas como subunidades e amostradas de acordo com a teoria 3-P. Outra aplicação inclue a amostragem para combustível nas pesquisas de energia de biomassa, amostragem de peso de animais silvestres, como o veado em estações de controle e outras. LUND (1975) mostra grande número de exemplos da aplicação de 3-P. O método pode ser aplicado quando a variável de interesse pode ser estimada e a partir daí alguns indivíduos apenas são medidos. Desde sua apresentação por GROSENBAUCH em 1963, a amostragem 3-P tem demonstrado ser um método eficiente de inventariar florestas, que deverá ser considerado como uma seria alternativa, mas com todos os outros métodos, seria pouco razoável esperar que sua aplicação pode ser generalizada para todas as circunstâncias .

5. LITERATURE

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