

Mathematical modeling in the initial years as a perspective for mathematics teaching: a brief history of Brazilian publications in journals (from 2009 to 2018)¹

*A modelagem matemática nos anos
iniciais como perspectiva para o ensino de
matemática: um panorama de publicações
brasileiras em periódicos (de 2009 a 2018)*

*La modelación matemática en los años
iniciales como perspectiva para la
enseñanza de matemática: un panorama
de publicaciones brasileñas en periódicos
(de 2009 a 2018)*

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ABSTRACT

This article presents an overview of scientific productions related to mathematical modeling in the initial years of elementary education in Brazil as a new perspective for Mathematics teaching concerning the period from 2009 to 2018. This research was developed according to the criteria

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of the qualitative research, using the descriptive and analytical methods and aspects of a research called state of the art. The process of collecting and recording data was carried out using audiovisual means and materials: research at online libraries pertaining to four journals of the Mathematical Education field, Mathematical Education Bulletin (BOLEMA - Boletim de Educação Matemática), Mathematical Education Study and Research Group (GEPEM - Grupo de Estudos e Pesquisas em Educação Matemática) of the Federal University of Rio de Janeiro (UFRJ), Mathematical Education Research (EMP - Educação Matemática Pesquisa) and Zetetiké, as well as the use of the Microsoft Office Word Software, for filtering and analyzing data during the investigative period. One the key objectives having been identified, they were developed by the research, methodologies and activities adopted, reaching the results found. The analysis of such articles allowed for the understanding of how mathematical modeling has been used as an innovative approach to teach this discipline and towards other areas of knowledge, revealing minimal scientific production on the subject and the need for incentives towards development of new research in the field.

Keywords: Mathematics Education. Mathematical Modeling. Initial Years of Elementary School. Journals. Overview.

RESUMO

Este artigo apresenta um panorama de produções científicas relativas à modelagem matemática nos anos iniciais do ensino fundamental do Brasil como nova perspectiva para o ensino de matemática referente ao período de 2009 a 2018. Esta investigação foi realizada segundo os critérios da pesquisa qualitativa, utilizando os caracteres descritivo e analítico e os aspectos de uma pesquisa denominada estado da arte. A coleta e o registro dos dados foram efetivados a partir de meios e materiais audiovisuais: pesquisas em bibliotecas on-line de quatro periódicos da área de Educação Matemática, Boletim de Educação Matemática (BOLEMA), Grupo de Estudos e Pesquisas em Educação Matemática (GEPEM) da Universidade Federal do Rio de Janeiro (UFRJ), Educação Matemática Pesquisa (EMP) e Zetetiké, bem como o uso do software do Microsoft Office Word para os filtros e análises no período investigado. Tendo sido identificados os objetivos, desenvolvidos pelas investigações, metodologias adotadas e atividades, chegando-se aos resultados encontrados. As análises dos artigos permitiram compreender como a modelagem matemática tem sido utilizada como uma perspectiva inovadora de ensino para essa disciplina e para outras áreas do conhecimento, revelando produções científicas mínimas no assunto tratado e necessidades de incentivo para o desenvolvimento de investigações na área.

Palavras-chave: Educação Matemática. Modelagem Matemática. Anos Iniciais do Ensino Fundamental. Periódicos. Panorama.

RESUMEN

Este artículo presenta un panorama de producciones científicas relativas al modelado matemático en los años iniciales de la enseñanza fundamental en Brasil como nueva perspectiva para la enseñanza de matemáticas referente al período de 2009 a 2018. Esta investigación fue realizada según los criterios de la investigación cualitativa, utilizando los métodos descriptivo y analítico y los aspectos de una investigación denominada estado del arte. La recolección y el registro de los datos se realizaron a partir de medios y materiales audiovisuales: investigaciones en bibliotecas en línea de cuatro periódicos del área de Educación Matemática: Boletín de Educación Matemática (BOLEMA), Grupo de Estudios e Investigaciones en Educación Matemática (GEPEM) de la Universidad Federal de Rio de Janeiro (UFRJ), Educación Matemática Pesquisa (EMP) y Zetetiké, así como por el uso de software, Microsoft Office Word para los filtros y análisis en el período investigado. Cuando se identificaron los objetivos desarrollados por las investigaciones, las metodologías adoptadas y las actividades, se llega entonces a los resultados encontrados. Los análisis de los artículos permitieron comprender cómo el modelado matemático se ha utilizado como una nueva perspectiva para la enseñanza para esa disciplina, y otras áreas del conocimiento, revelando producciones científicas mínimas en el tema tratado y la necesidad del incentivo para el desarrollo de investigaciones en el área.

Palabras clave: Educación Matemática. Modelado Matemático. Años iniciales de la Enseñanza Fundamental. Periódicos. Panorama.

Introduction

The first proposals and ideas to make use of mathematical modeling in Education began around 1970s in several countries. For example, projects led by Hans Freudenthall in the Netherlands, called IOWO (BIEMBENGUT, 2014). Also, it is possible to recognize others works by “Bernhelm Booss and Mogens Niss, at Denmark (Roskilde University), led [them], in 1978, to organize a conference on Mathematics and Reality, which contributed to the consolidation [1983] of the International Community of Teachers of Mathematical Modeling and Applications – ICTMA” (BIEMBENGUT, 2014, p. 15), leading to the introduction and propagation of studies or research on mathematical modeling in the form of articles, dissertations and theses in graduate programs *lato sensu* and *stricto sensu*, presented at international and national congress. With this, there are several academic papers in the context of mathematical education

to suggest the implementation of mathematical modeling in basic education (BARBOSA, 2003; ALMEIDA; SILVA; VERTUAN, 2013; BIEMBENGUT, 2014; SOARES, 2012a, 2012b, 2017, CRISTOVÃO; ALENCAR; BARROS, 2018) which focus on learning.

Among the concepts of modeling, initially, the following is emphasized:

The expression *modeling* can be treated as a synonym of *molding* [...] aiming to investigate, develop and transform a situation or a problem from reality through research and studies, analyzes, forecasts, simulations, modifications and explanations that favor the development of mathematical concepts (SOARES, 2017, p. 46, highlights by the author).

According to Biembengut (2014, p. 21), “Modeling is the process involved in preparing [use and validation] a model in any field of knowledge. It is a research process”. At the same time, in the conception of Almeida, Silva and Vertuan (2013, p. 17) it “is an educational alternative in which we approach, through mathematics, a problem-situation which is not essentially mathematical”. In addition, to Barbosa (2003, p. 68-69), modeling is “understood as a learning environment, which is associated with questioning and research”. “The first refers to the act of creating questions and/or problems while the second, is the search, selection, organization and manipulation of information and reflection over it” (BARBOSA, 2003, p. 68-69). Thus, “Both [the] activities are not separate, but articulated in the process of involving students to address the proposed activity. In it, one can raise questions and conduct investigations that affect the scope of reflective knowledge” (BARBOSA, 2003, p. 69). Despite the many different conceptions, researchers agree that a particular activity, task or modeling work, in general, has as its starting point a theme, situation, problem or extracted phenomenon from reality. Moreover, there are different perspectives of modeling concepts and not all can be used at the same level of education or for the same purpose (SOARES, 2012a, 2012b, 2017; PARRA-ZAPATA; VILLA-OCHOA, 2015, CARMONA-MESA; SALAZAR; VILLA-OCHOA, 2018, SILVA; BUENO, 2018).

In the face of different conceptions of mathematical modeling and the inability to integrate them in the same way in all curricula, it is important for the international community to get to know the various research in development, including those from Brazil, since the field of mathematical modeling shows a high degree of development and consolidation. With this interest, Madruga and Breda (2017) used the educational mapping methodology to account for

their theoretical frameworks, investigated problems, research interests and methodologies, bringing the main results and contributions of ten researches developed in Brazil to advance the theme in the field and the prospects for continued study. However, this mapping was not focused on the understanding of models and modeling in basic education, it considered: proposal, environment, method and alternatives. Recognizing these understandings is important both for research and for teachers. Therefore, in this article, there was a concern to present an overview of the scientific production developed by researchers in the field, in order to support future research students interested in the subject relating to mathematical modeling towards teaching in the Initial Years. And so, we present the results obtained in response to the following question: *What insights can model and modeling present in the researches developed in Brazil throughout the last decade?*

From this period, we considered the publications of the last decade published in four journals of the Mathematical Education field, as will be clarified in the methodology section.

Methodology

To achieve our goals, a qualitative research was conducted (CRESWELL, 2010, 2014; SEVERINO, 2007), focused on descriptive and analytical characters (FIORENTINI; LORENZATO, 2012; SEVERINO, 2007) and aspects of a research called *state of art* (FERREIRA, 1999, 2002; FIORENTINI, 1994; SOARES, 2017) to present an overview of some of the scientific productions related to mathematical modeling in the Initial Years, from 2009 to 2018, i.e. the last decade.

In this sense, one can say that this overview aimed to present a general picture of the extent of the studies or research regarding the theme chosen for a limited period, without covering large volumes of data collection.

As for the procedures for collecting and recording data, these were recovered from media and audiovisual materials: research in *websites* of four journals from the Mathematical Education field in Brazil: *Mathematical Education Bulletin* (BOLEMA - Boletim de Educação Matemática), *Mathematical Education Study and Research Group* (GEPEM - Grupo de Estudos e Pesquisas em Educação Matemática) of the Federal University of Rio de Janeiro, *Mathematical Education Research* (EMP - Educação Matemática Pesquisa) and *Zetetiké*. We selected these magazines because, according to Fiorentini and Lorenzato

(2012), they address the study and research related to mathematical education. We also made use of *Microsoft Office Word* Software, for organizing the collected data and analysis of the requested investigations. For this, it was necessary to resort to Soares (2017) to guide the collection and selection of items in *online libraries*, as shown in the following board:

BOARD 1 - CRITERIA AND ELABORATED PROCEDURES TO LIMIT THE SIZE OF THE SAMPLE OF SCIENTIFIC PRODUCTIONS ON MATHEMATICAL MODELING IN THE INITIAL YEARS OF BASIC EDUCATION (2009 TO 2018)

Criteria and elaborated procedure:

It is necessary to study the following keywords and terms, inserted or not, on titles and unique abstracts of scientific productions: *approach, alternative (educational) environment, strategy (teaching), modeling, mathematical modeling, molding, mathematical molding, mathematical education, mathematics, mathematical model(s), model(s), proposed (teaching) method and methodology*, which are originated from fundamental literature about mathematical education and modeling according to the years of occurrence, such as: D'ambrosio (1986), Borba (1987), Burak (1992), Barbosa (2001), Brasil (2006), Bassanezi (2009), Beltrão and Iglori (2010), Herminio and Borba (2010), Klüber and Burak (2012), Silveira and Caldeira (2012), Almeida, Silva and Vertuan (2013), Biembengut and Hein (2014), Soares and Iglori (2016), Smith and Santos Junior (2016) and Soares (2017). Moreover, it is necessary to read, analyze and interpret, critically and reflectively, whole summaries of academic research and, when necessary, their original texts partially or in its entirety. Still, it is essential to investigate whether there is any academic research on modeling in mathematical education that displays hidden keywords, i.e., a particular scientific production of this nature that, however, does not expose such terms mentioned above for the effectiveness of research engines.

SOURCE: Soares (2017, p. 93, with adaptation).

According to Soares (2017), this criteria and elaborated procedure was important in order to consider the following keywords to the search of scientific production on mathematical modeling in the Initial Years: *from first to fourth grades, Initial years, Elementary School, Elementary School I, basic education and basic teaching*. Therefore, during the collection and data registration processes, the samples related to scientific productions that have not studied and/or researched modeling in the initial years were not considered, specifically, resulting in three articles, as explained in the following table:

TABLE 1 - SCIENTIFIC PRODUCTIONS IN MATHEMATICAL MODELING AND MATHEMATICAL MODELING IN INITIAL YEARS OF ELEMENTARY SCHOOL (2009 TO 2018)

Journal Names	Scientific Productions on Mathematical Modeling	Scientific Productions on Mathematical Modeling (%)	Scientific Productions on Mathematical Modeling in Initial Years of ES*	Scientific Productions on Mathematical Modeling in Initial Years of ES* (%)
BOLEMA	27	96,43	1	3.57
GPEM	6	85.71	1	14,29
EMP	27	100	0	0
ZETETIKÉ	5	83.33	1	16.67

*Elementary School

SOURCE: The authors (2019).

In Table 1, of the four journals that were selected for the pursuit of this scientific work, it can be noted the presence of papers on mathematical modeling in all educational segments. However, three empirical and/or bibliographical investigations were obtained on the subject of mathematical modeling in the initial years of Elementary School. In one of the journals, specifically EMP, we found no research involving this issue and this level of education, which is a warning on the incentive of research publications in this journal's field.

Furthermore, the data in Table 1 was extracted and analyzed from the following sources, as elucidated in Board 2:

BOARD 2 - ADDRESSES OF THE ONLINE LIBRARIES OF THE JOURNALS WITH SCIENTIFIC PRODUCTIONS ON MATHEMATICAL MODELING AND MATHEMATICAL MODELING IN THE INITIAL YEARS (2009 TO 2018)

Journal Names	Online Libraries	Access dates
BOLEMA	A) http://www.periodicos.rc.biblioteca.unesp.br/index.php/bolema B) http://www.scielo.br/scielo.php?script=sci_serial&pid=0103-636X&lng=pt&nrm=iso	Feb. 23, 2019
GPEM	http://www.ufrj.br/SEER/index.php?journal=gepem&page=login&source=%2FSEER%2Findex.php%3Fjournal%3Dgepem	Feb. 24, 2019
EMP	https://revistas.pucsp.br/emp	Feb. 24, 2019
ZETETIKÉ	https://periodicos.sbu.unicamp.br/ojs/index.php/zetetike/index	Feb. 23, 2019

SOURCE: The authors (2019), data extracted and analyzed from the journals.

For the analysis of scientific production obtained in online libraries, the following criteria was used, presenting in this manner:

- a) Author and year of publication;

- b) Title;
- c) Purpose and methodology of the article;
- d) Mathematical concepts carried out, if any;
- e) Summary of conclusions.

Results and discussions

Based on Table 1 and Board 2, although the research has been limited by the period of 2009 to 2018, it can be said that only from 2014 there was scientific productions published on mathematical modeling in the initial years of Elementary School, as clarifies Board 3. Thus, the concern with investigations in this subject for such teaching segment is recent and is presented as a new methodological approach to the teaching of Mathematics:

BOARD 3 - SCIENTIFIC PRODUCTIONS ON MATHEMATICAL MODELING IN INITIAL YEARS (2009 TO 2018)

Periodicals	Authors	Year	Titles
GEPEM	- Elizabeth Gomes Souza; - Ana Virginia de Almeida Luna; - Larissa Borges de Souza Lima.	2014	The role of the teacher of initial years in the production of children's discourses in mathematical modeling activities
BOLEMA	- Helena Gil Guerreiro; - Maria de Lourdes Serrazina.	2017	Learning rational numbers with understanding involving an Emerging Modeling Process
ZETETIKÉ	- Luzinete de Oliveira Mendonça; - Celi Espasandin Lopes.	2017	Reflections on the pedagogical action in the development of mathematical modeling

SOURCE: The authors (2019), data extracted and analyzed from periodicals.

Among the scientific productions collected from 2009 to 2018, there is the work of Souza, Luna and Lima (2014), which aimed to “analyze how the teacher plays his guiding role on the discourse created by the children in an mathematical modeling activity in the Initial Years” (SOUZA; LUNA; LIMA, 2014, p. 34). The methodology used in this investigation was the discursive production of the investigated students and teachers, having, as theoretical support for analysis, the studies of Anna Sfard. The data collection was carried out through observation, with audio recording followed by transcription and analysis. Thus, this study examined a teacher and twenty-two students of

the fourth grade of Elementary School. The authors used Charmaz (2006) to perform the discourse analysis of the student's reports. The theme addressed to the mathematical modeling activity was "virtual water" and the main teacher of the analyzed group started the activities with the presentation of a video on the theme. Among the discourses, we highlight the following ones in Board 4, where *TCHR* means *Teacher* and *C* means the *Child* analyzed:

BOARD 4 - DISCOURSE I

[18] TCHR: After the video we watched, what do you think "virtual water" really is?

[19] C2: Every water we consume.

[20] TCHR: Every water we consume where? What is virtual water?

[21] C4: It's the water that we drink, and from the sink, you can use it to wash the car.

[22] TCHR: Only this water?

[23] C6: To make a license plate, and many other things, to make food...

[24] TCHR: And, to mix in cement for our homes or for industry?

[25] C5: For the Industry.

[26] TCHR: How?

[27] C1: To make products: rubber, cement, glass, everything that comes from Industry.

SOURCE: Souza, Luna and Lima (2014, p. 39).

The authors, presented with this discourse, were able to perceive the students' understanding of "virtual water" and expanded their reflections. Then, the teacher presented a text on what "virtual water" would be. Next, still, the following discourse took place, as shown in Board 5:

BOARD 5 - DISCOURSE 2

[63] TCHR: The text we just read says that Brazil exports soybeans to another country and that during the whole year we spent 45 million cubic meters of water.

[64] C2: Hey Teach, what does cubic mean?

[65] TCHR: It's a measure, for example, the depth of a pool.

[66] C6: Here on the internet, Teach, a pool - those athletic pools Gustavo Borges swims on, it said it has 2,700 cubic meters of water.

[67] TCHR: Let's make this calculation? I want to know how many times does 2,700 fit inside those 45,000,000.

[68] C6: Is that a multiplication or a division?

[69] TCHR: How many times 2,700 can fit into 45,000,000? What is the strategy?

[70] {A child grabs a calculator and makes the division. It results in 16,373}. 40

[71] TCHR: That's right. Guys, look at what we found out... so for all the soybeans Brazil produces in one year, how many pools do we need?

[72] C7: 16. 373. [...]

[77] TCHR: Is this a little or a lot?

[78] CHLDN: A lot.

SOURCE: Souza, Luna and Lima (2014, p. 39-40).

Based on Board 5, it was possible to see the amount of cubic meters of water that Brazil spends in a certain agricultural production, which was identified by the work of the teacher, who instigated the knowledge of the students towards the decimal numbering system and unit measures.

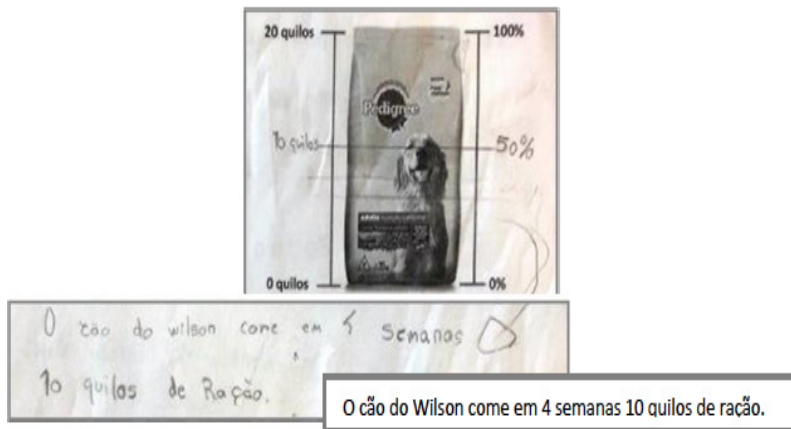
In this context, with the focus on the cubic meters, the teacher allowed the students to do comparisons between athletic pools and the amount of water used to estimate if it was a large or small quantity. After that, the teacher asked students to reflect on the impact of water usage, focusing on the water used in the three daily meals. With this inquire, the teacher could instigate students to think: "Each one of you, every family, eats around 26 kg of meat in a week. Now I want you to calculate how many liters of virtual water your family uses

each week. 1 kg of meat = 13,000 liters. So, let's estimate, then I'll go back to check the work" (SOUZA; LUNA; LIMA, 2014, p. 41). This caused the students to start researching how many liters of water they used without knowing, from the production of food, proposing the construction of graphs to perform numeric comparisons. From these investigations, the subjects reached the cycle of consumption and production of food and could, in the end, understand what "virtual water" really is.

Still in that decade, between the scientific papers obtained on mathematical modeling in the initial years, the article from Guerreiro and Serrazina (2017, p. 182) will be highlighted, whose "goal [is] to see how we can build learning with understanding of rational numbers." Thus, it is noted: "In this article, we aim to understand the role that representations take as they are used and transformed as models of contextualized situations, for students of the 1st cycle, and how they evolve the models of reasoning" (GUERREIRO; SERRAZINA, 2017, p. 182), which involved students from eight to ten years old. Methodologically, we "used a qualitative approach with interpretative focus recurring to a research based on *design*, relying on the construction and implementation of a teaching experience in the classroom" (GUERREIRO; SERRAZINA, 2017, p. 186), allowing the teacher to work with tasks dealing with issues extracted from the reality of the students.

Among the tasks performed, there is the one that addresses percentage, as highlighted by Guerreiro and Serrazina (2017) in the definition of the modeling problem and its resolution: the search task to "relate the mass to the percentage of the amount of feed in the bag, an interpretation of percentage as a comparison scheme, which refers to the construction, in an initial stage, of the reasoning appealing to the intuition concerning the ratio" (GUERREIRO; SERRAZINA, 2017, p. 194). This way, "The imagery has given an iconic representation of the feed bag supported by two numeric lines, representing two scales, [...] where the top of the bag represents the bag full, with 20 kg of feed, indicating 100%" (GUERREIRO; SERRAZINA, 2017, p. 194), as shown in Figure 1:

FIGURE 1 – PERCENTAGE DEVELOPMENT TASK (GROUP X)

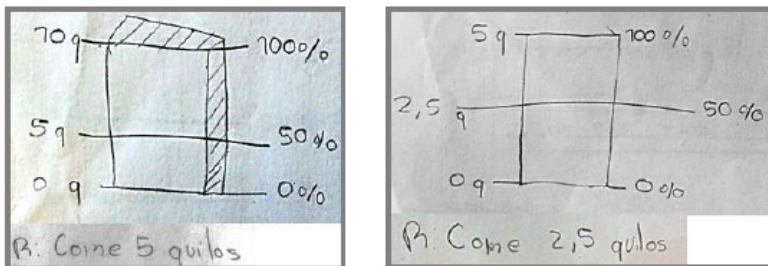


SOURCE: Guerreiro and Serrazina (2017 p. 194).

In this task, “The presence of these two scales allows for establishing comparison relationships, varying the feed mass of a bag according to the amount of feed that the bag carries, in order to induce the construction of meaning in the use of the double numeric line” (GUERREIRO; SERRAZINA, 2017, p. 194). Mathematical modeling in the initial years provides investigation, discussion and transformation of the real-life phenomena and problems of everyday life into mathematical models, aiming to develop mathematical learning. It seeks to investigate and express part of reality through a mathematical relationship exploiting the skills and knowledge of the students.

Thus, based on Figure 1, “Every group estimates 50% as half the mass of feed in kilograms, as suggested, without difficulty. Some student groups use the iconic representation given directly” (GUERREIRO; SERRAZINA, 2017, p. 194). And more, “Other groups make use of the given image, but rebuilding it, modeling the situation to support the construction of relations” (GUERREIRO; SERRAZINA, 2017, p. 194-195), highlighting the “relationship between quantities, supported by two lines, that the group uses to build their reasoning. In each point of the task, the group refers to the reconceptualization of the unit” (GUERREIRO; SERRAZINA, 2017, p. 195), as Figure 2 clarifies:

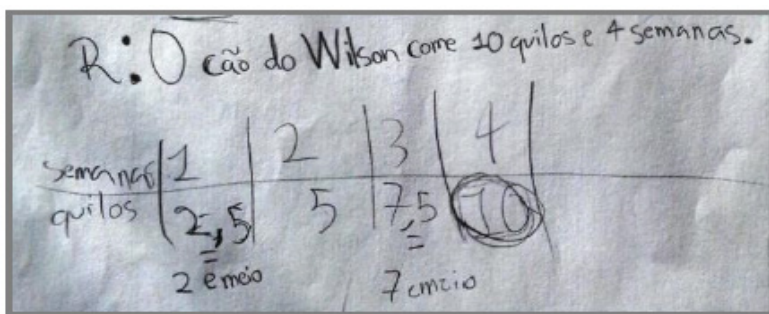
FIGURE 2 - PERCENTAGE DEVELOPMENT TASK (group Y)



SOURCE: Guerreiro and Serrazina (2017 p. 195).

The modeling allows the development of mathematical models through symbols, mathematical structures and relationships in accordance with the characteristics of a situation, phenomenon, or through data extracted from reality, in which you create, explore and solve problems by exploring general skills of the subjects. Thus, from Figure 2, “yet another group, having calculated the percentage corresponding to 50% and 25%, students choose to model the situation through a table of reason [...] which allows them to also interpret the relationship as a function of time.” (GUERREIRO; SERRAZINA, 2017, p. 195), as explained in Figure 3:

FIGURE 3 - PERCENTAGE DEVELOPMENT TASK (group Z)



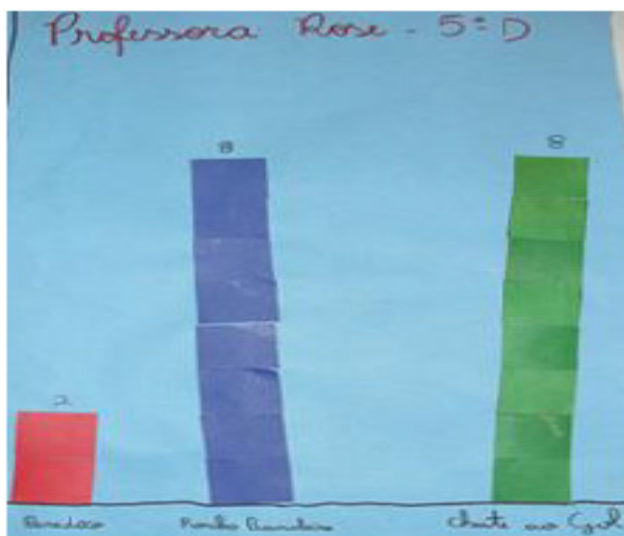
SOURCE: Guerreiro and Serrazina (2017 p. 195).

For Guerreiro and Serrazina (2017), students managed to interpret constructs of less common rational numbers in the initial years, as the measure and the reason, according to the need to involve various constructs in an initial work with the percentage, leveraging the advantages of possible representations and analyzing its limitations.

From that time, it's worth mentioning the work of Mendonça and Lopes (2017), which “aimed to improve understanding about the pedagogical action in learning environments from the perspective of mathematical modeling” (MENDONÇA; LOPES, 2017, p. 305). Therefore, “It was conducted a qualitative research. We've taken, as the object of analysis, actions and dialogues held in a specific context of teaching and learning from a teacher and her students, a 5th grade class of a municipal school” (MENDONÇA; LOPES, 2017, p. 305), involving twenty students and making use of case study, descriptive procedures and assumption of subjective inference of researchers for the development of statistical concepts.

Mendonça and Lopes (2017) go on to explain that the statistical activities performed dealt with a vertical column chart made with squares of paper pasted on the cardboard, and these activities belong to the “Games” Project. From this, the authors pose a modeling problem: “The column chart, built with paper squares (representing the students' votes), presents the three most voted games and the number of votes” (MENDONÇA; LOPES, 2017, p. 315). “From the origin, on the horizontal axis, there are three columns: wall target (2), capture the flag (8) and goal kicks (8). The vertical axis is numbered” (MENDONÇA; LOPES, 2017, p. 315), as Figure 4 clarifies:




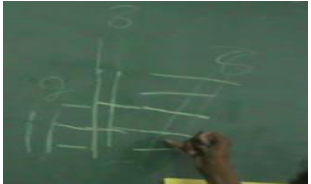
FIGURE 4 - CHART OF GAMES PREFERRED BY THE 5TH GRADE - D (group 1)



SOURCE: Mendonça and Lopes (2017, p. 315).

In this development, a group presented a mathematical model involving data collected in a 5th grade class. “These results were revealed by a vertical column chart made with pieces of paper glued to the cardboard, but the columns were tilted” originating “a discussion where the teacher sought to lead the students to realize the necessity of the perpendicularity of the columns, relative the horizontal axis” (MENDONÇA; LOPES, 2017, p. 315). Therefore, certain graphic representations were explored on a blackboard as elucidated in Board 6:

BOARD 6 - REASONINGS AND GRAPHIC REPRESENTATIONS CARRIED OUT BY THE STUDENTS

Modeling situations	Analysis
	<p>Here, the student wanted to convince her listeners that the lines and the boxes allows the counting, even with the lack of numbers in the columns, which furthers our understanding of the chart. So once again she justified the tilted columns in the chart.</p>
	<p>With her words and with the representation, the student tries to explain that the number of the column above is enough for understanding the chart. That is, the columns don't need to be perpendicular and, by its record, the axes aren't needed.</p>
	<p>Here, the student wanted to convince her listeners that the lines, and the boxes, allows the counting, even with the lack of numbers in the columns, which furthers our understanding of the chart. So once again she justified the tilted columns in the chart.</p>
	<p>The student stretches the lines previously made in the first two columns to the third. It seems as she hasn't yet been satisfied of the need for the column to be perpendicular to the horizontal axis. She counts the spaces between the lines drawn on the tilted column, totaling six, as she considered the space after the last line, up and down (in the picture, the space indicated by the finger was the second in her count).</p>

SOURCE: Mendonça and Lopes (2017, p. 316-318).

In the first modeling scenario, a student's reasoning is revealed when trying to understand and express the construction of the graphics, while, in the following situation, the said student affirms that the columns of the graphics don't need to be perpendicular. In the third situation, the student explains the counting process by using lines and squares without worrying about the inclined chart, whereas, in the latter modeling situation, she concludes stretching, horizontally, the lines of the prepared columns.

Mathematical modeling as a strategy allows us to enhance the teaching and learning of mathematical concepts based on the inquiry process, cooperation and interaction between teacher and students, enabling work in a group, in a dynamic way and with different types of mathematical resolutions to real problems.

For Mendonça and Lopes (2017), mathematical modeling in the classroom requires a reflective, committed and active attitude from the teacher, along with various degrees of intervention in order to contribute to the development of concepts, attitudes and arguments, taking into account autonomy for the construction of knowledge by the subjects.

The Mathematical Modeling as a new approach to learning: some considerations

Throughout this article, the question posed was answered and the objective was reached by presenting an overview of scientific works relating to mathematical modeling in the initial years of Elementary School as a new perspective to teaching Mathematics for the period 2009-2018, last decade of this century, in Brazil, based on four journals from the Mathematical Education field: *BOLEMA*, *GEPEM*, *EMP* and *Zetetiké*.

In this conclusion, a few essential considerations are necessary to clarify implications about the analysis of the potential of the mathematical modeling methodology in the initial years of elementary school.

In this sense, mathematical modeling is seen as a new approach to learning in the initial years because it works with a sociocritical vision that allows students to be participants in the development of their learning. Thus, subjects develop a creative capacity to solve the problems. With that, mathematical modeling provides critical education through research and real problems resolutions using Mathematics.

Therefore, the overview presented revealed that there are minimal scientific publications about mathematical modeling in the initial years of Elementary

School, indicating the need for further studies or research that address this theme in this level of education.

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