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The teaching of Mathematics in the thinking of Comenius, Pestalozzi and Montessori[1]

O ensino de Matemática no pensamento de Comênius, Pestalozzi e Montessori

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ABSTRACT

In this article, we discuss the fundamentals for the teaching of Mathematics in Traditional Pedagogy and New Pedagogy, markedly, based on theories of Comenius, Pestalozzi and Montessori. This research is documental and bibliographic. At the end of the analysis, we conclude that: there is a circulation of ideas between the educational thoughts of Comenius, Pestalozzi and Maria Montessori regarding the use of didactic materials and that in the methods proposed by these thinkers Mathematics is associated with practical and gradual activities.

Keywords: Fundamentals of Education. Mathematics. Comenius. Pestalozzi. Montessori.

RESUMO

Neste artigo, discutimos os fundamentos para o ensino da Matemática na Pedagogia Tradicional e na Pedagogia Nova, marcadamente, a partir das teorias de fronteira de Comenius, Pestalozzi e Montessori. Esta pesquisa é de cunho documental e bibliográfico. Ao final da análise, concluímos que: há uma circulação de ideias entre o pensamento educacional de Comenius,

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Pestalozzi e Maria Montessori no que diz respeito ao uso de materiais didáticos e que nos métodos por estes pensadores a Matemática está associada às atividades práticas e gradativas.

Palavras-chave: Fundamentos da Educação. Matemática. Comenius. Pestalozzi. Montessori.

Introduction

In Brazil, the biennium 2017 and 2018 was called the "Biennium of Mathematics", since the country hosted the International Mathematical Olympiad, the International Mathematics Congress and that the National Council for Scientific and Technological Development (*Conselho Nacional de Desenvolvimento Científico e Tecnológico,* CNPq) defined as the central theme of the Science and Technology Week of 2017 as "Mathematics is in Everything".

Faced with such protagonism, as professors who educate Mathematics teachers or Pedagogues attentive to the historical and philosophical foundations of education, we set out to discuss elements for the teaching of Mathematics in Traditional Pedagogy and New Pedagogy,[2] markedly from the frontier theories of Comenius, Pestalozzi and Montessori,[3] in an attempt to reflect and answer the questions: "How is mathematics taught?"and, "which mathematics is taught?"

The justification for choosing these thinkers refers to their interface between different ways of thinking about teaching. Comenius being linked to Traditional Pedagogy, considering that he corroborated and experienced changes in the school context and the teaching habits in action. So, there was an increase of "[...] the number of schools that welcome more children, whose presence is a little more frequent. The masters create new know-how to solve daily problems. This codified know-how affects not only the content to be taught but also, all aspects of classroom life"[4] (GAUTHIER, 2014, p. 155).

Then, Pestalozzi and Montessori constitute themselves as agents of the process of a New Pedagogy, which "[...] therefore becomes, for these authors of

2 According to Gauthier (2014) the pedagogy, whose birth would have occurred in the 17th century, kept itself relatively stable until the 19th century, both in its spirit as in its practices, to constitute a kind of pedagogical tradition in Europe. It is in this context that traditional pedagogy situates itself and, in a stricter manner, the new pedagogy from the 19th century henceforth.

3 During the text we present and discuss ideas from the referred thinkers of education.

4 Such considerations refer to the simultaneous teaching, codes oriented to posture, the dislodgements, the punishments, the place of each student in the classroom etc.

the end of the 19th and beginning of the 20th century, not only a science, but a science applied, whose fate is linked to the state of progress of the fundamental knowledge in psychology" (GUATHIER, 2014, p. 165).

According to Gauthier (2014), pedagogy, whose birth would have occurred in the 17th century, remained relatively stable until the 19th century, both in its spirit and in its practices, to constitute a kind of pedagogical tradition in Europe. It is in this context that Traditional Pedagogy and, in a stricter manner, New Pedagogy[5] are located from the 19th century onwards, printing new conceptions of teaching.

Thus, based on the interrelation of the theories of Comenius, Pestalozzi and Montessori with contemporary authors in the area of Education and Mathematical Education, we reflect on the teaching of Mathematics in Traditional Pedagogy and New Pedagogy.

A look at the teaching and Mathematics in Comenius

João Amós Comenius (1592-1671), which we will call Comenius, was born in Moravia (region of the current Czech Republic) and lived in the 17th century, which was considered by historians, a tragic, confused, problematic century, but which recommended transformations, including, concerning Pedagogy, since, from Comenius

> [...] for the first time, some of the already relevant problems of Pedagogy are outlined in an organic and systematic way: from the anthropological-social project that should guide the teacher to the general and specific aspects of didactics, to arrive at educational strategies that reference to various instructional guidelines (CAMBI, 1999, p. 281).

Cambi (1999) tells us that Comenius initially wrote texts of a publicity nature, with the aim that the Czech people would learn and know their history, however, when advancing in the writings, he wrote several works that turned to the importance of the educational process, since he considered education as a "way to salvation, an education for eternal life" (COMENIUS, 1966).

5 For more information see Cousinet (1959).

Comenius was forced to leave his homeland due to the political and religious struggles that took place in Moravia, so that in the period when he was exiled – 1628 to 1632 – he wrote the *Czech Didactics*, which, when translated into Latin, in 1957, received the name of *Magna Didactics*, seeking through this, to build "a universal science capable of producing that general pacification among men that constitutes the ultimate aspiration of his life" (CAMBI, 1999, p. 286).

With *Magna Didactics*, Comenius systematized a strictly pedagogical knowledge. According to Boto (2019), in this work Comenius intended to understand the learning path, suggesting that schools organize their goals, their discipline and their learning method, as well as the teaching ritual. Thus, Comenius built his educational project, becoming the first systematizer of pedagogical discourse, organically related to the technical aspects of training with a reflection on man (CAMBI, 1999).

Comenius presented a method that is directly inspired by the Cartesian precept, according to which each stage of the learning process must suppose the chain of contents, starting from the simple up to the complex, from the general to the particular. And, teaching would take place with students divided into separate classes in the classroom according to the criteria of each student's level of learning (BOTO, 2019).

In his method, Comenius recommended the teacher as a central figure, since it would be up to the students to listen and obey, this being a "mechanically programmed education", but also a pedagogy concerned with communication, being effective, natural and intuitive. Although Comenius's ideas regarding the psychological understanding of the teaching and learning process were outlasted, they were considered an advance for the time, for, grandly, proposing a school for all, even in the face of the domination of the Catholic Church, that, as we know, in the period known as *drought*, there was a return to the formation of the ruling classes (NARODOWSKI, 2001).

Furthermore, it is worth saying that he

> [...] advances a proposal for a school organization that provides for four successive degrees, for each one of which outlines the objectives, contents and methods, with meticulousness and an excessive minuteness, which lead to repetition and pedantry (CAMBI, 1999, p. 290).

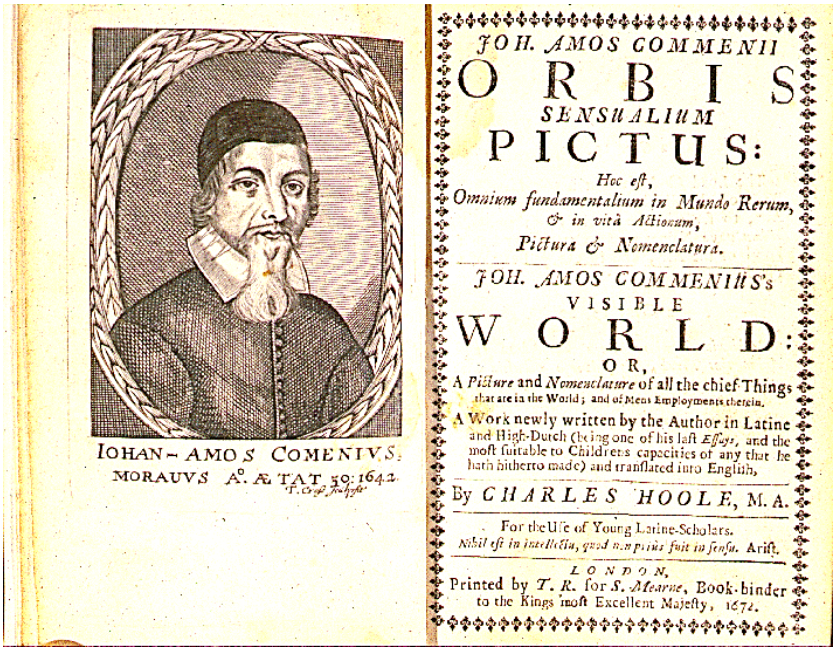
In this sense, Narodowski (2001, p. 56) expresses that, in *Magna Didactics* Comenius offers a methodological scheme by which "[...] it will be possible to educate the youth and by which it will be possible to have a school that until

then had not been". Comenius being interested in "[...] standardizing the indispensable elements that will constitute the model sought; and his interest is that such a model is internally harmonious. Order achieved through rationality is the guiding idea; this has as its purpose the good disposition of the elements" (NARODOWSKI, 2001, p. 56).

This time, teaching everything to everyone was configured as the Comenian utopia which had some didactic prescriptions, for example: a) clarity; b) go from the general to the particular; c) gradual advance; d) learn what is useful. In this utopia, we ask: what is the place of Mathematics?

Comenius, in *Magna Didactics*, pointed out the importance of using different materials in Mathematics classes, including recommendations for building models to teach Geometry. And, in 1650, Comenius published the book *Orbis Sensualium Pictus*, from which he suggests the study of different themes based on the use of images.

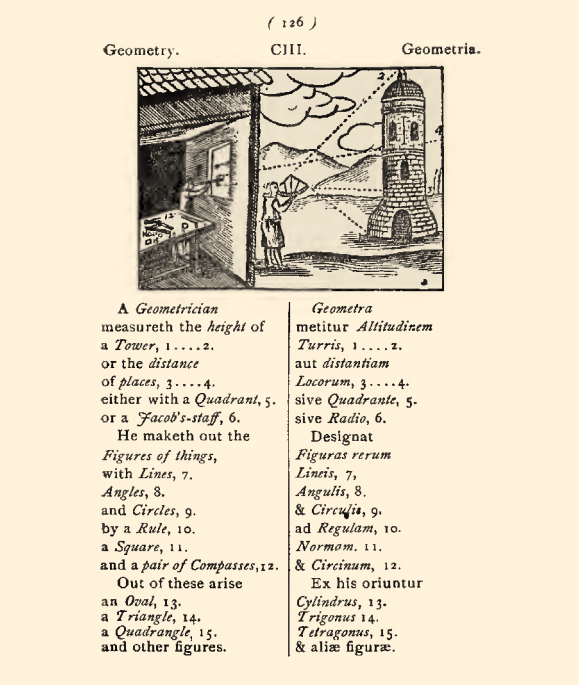
FIGURE 1 – ORBIS *SENSUALIUM* PICTUS, BY COMÊNIUS (1658).



SOURCE: Available on: https://archive.org/stream/cu31924032499455#page/n165/mode/2up/search/arit. Access on: jan. 4, 2019.

In the book Orbis *Sensualium* Pictus[6], Comenius devoted one of the lessons to the study of geometry. In this lesson, there is an indication to study the definition of Geometry. The contents about lines, circles, quadrants, triangles, squares and other figures are presented based on association with reference numbers in a figure.

FIGURE 2 – GEOMETRY LESSON IN THE BOOK *ORBIS SENSUALIUM PICTUS*, BY COMÊNIUS (1658).



(126)

Geometry. CIII. Geometria.

A *Geometrician* measureth the *height* of a *Tower*, 1....2. or the *distance* of *places*, 3....4. either with a *Quadrant*, 5. or a *Jacob's-staff*, 6. He maketh out the *Figures of things*, with *Lines*, 7. *Angles*, 8. and *Circles*, 9. by a *Rule*, 10. a *Square*, 11. and a *pair of Compasses*, 12. Out of these arise an *Oval*, 13. a *Triangle*, 14. a *Quadrangle*, 15. and other figures.

Geometra metitur *Altitudinem Turris*, 1....2. aut *distantiam Locorum*, 3....4. sive *Quadrante*, 5. sive *Radio*, 6. Designat *Figuras rerum Lineis*, 7, *Angulis*, 8. & *Circulis*, 9. ad *Regulam*, 10. *Normam*. 11. & *Circinum*, 12. Ex his oriuntur *Cylindrus*, 13. *Trigonus* 14. *Tetragonus*, 15. & aliæ figuræ.

SOURCE: Available on: https://archive.org/stream/cu31924032499455#page/n165/mode/2up/search/arit. Access on: jan. 4, 2019.

In figure 2, there is the presentation of a geometer analyzing drawings and geometric calculations in a workshop and a subject in an external environment making use of theodolite to measure angles towards a tower. Still, in figure 2, we

6 "Orbis Pictus", created in Sárospatak, in Hungary, 1657 and published in 1658, in Nuremberg, is considered the first illustrated didactical book and the first spelling book of the christian western world. It was used in reformist Europe for more than two centuries after its publication (CAMBI, 1999).

realize that the world around men is valued by Comenius during mathematical learning, as the observations made to obtain angle measurements or to perceive geometric shapes are made through curiosity, research, admiration, experience (as an experiment).

This understanding is in line with the ideal of teaching everything to everyone, as this ideal consists of teaching under the rational parameters of ordination with the ambition to equate the contents destined to the transmission of knowledge in the concrete space of the school. For Comenius, everything that is in the visible world is convenient for the school to educate man for this. And, in this sense, geometry was used to read the world, since it is considered the contents of lines, circles, quadrants, triangles, squares and other figures present in the world of men.

Thus, the mathematical elements would corroborate the principle of transferability, defended by Comenius, which corresponds to the general rule of forming man in the foundations of the main things that exist and are created.

And, considering Comenius' contributions to the foundations for the teaching of Mathematics in the context of the ideas of Traditional Pedagogy in the next sections we will discuss the ideas of Pestalozzi and Montessori concerning the teaching of Mathematics in New Pedagogy.

A look at teaching and mathematics in Pestalozzi

Johann H. Pestalozzi (1746-1827), born in Zurich, Switzerland, had his thinking marked by the influence of Jean-Jacques Rousseau[7] and the valorization of nature, seeing the activity in education as being one of the best tools so that the child could show its full potential[8] (ARCE, 2002).

Pestalozzi's educational thinking is associated with his experiences as an educator with the creation of schools, from which he outlined his teaching

7 For Gauthier (2014), Jean-Jacques Rousseau was one of the main thinkers of the XVIII century and his influence is still felt in our days. His educational conception is functional, implicating a vision exclusively utilitarian of culture: the child must only learn that which will be directly useful during their existence.

8 In the anxiety to spread his method, it drove Pestalozzi to write a series of letters and books. From which stand out: Leonard und Gertrud (1781); How Gertrud teaches her children (1801); *Swansong* (1827).

method with the interconnection between the hand, the heart and the brain. Thus, the principles of the method refer to:

> 1. First, any true reform must begin with the individual and not with society. [...];
> 2. Second, the individual can only be elevated by giving him the power to become self-sufficient. [...] the best service that society can provide for the individual is to teach him, first, to respect himself, and second, to support himself;
> 3. Third, the only way to achieve the desired end is through development. The seeds of independent action, latent in every child, are just waiting for an opportunity to grow. Education must offer the opportunity for it to grow (EBY, 1962, p. 383-384).

Tröhler (2014) discusses the educational experiences carried out by Pestalozzi and the organization of his method and points out that he valued the teaching of languages so that children could verbalize their impressions of the world. It also highlights that Pestalozzi gave more theoretical importance to professional training and more practical importance for elementary moral education. Thus, Pestalozzi defined his method and established that the best education is that which accompanies the child's evolutionary development.

Pestalozzi, in Aargau, installed the educational center called Neuhof (1770-1780) for the education of poor children who also worked in the spinning and weaving of cotton. Later, Pestalozzi transformed the castle of Yverdun, in Neufchâtel, in French Switzerland, into a center for experimentation and construction of his educational ideas (OLIVEIRA, 2017).

For Eby (1962), each education center founded by Pestalozzi was an "experimental school". The two main institutions that made him famous were the institute in Burgdorf, from 1800 to 1804, and another similar one in Yverdun, from 1805 to 1825.

Considering these experiences, Pestalozzi, in a letter of October 16th, 1821, in Yverdon, expresses that

> All the true inner strength of thought and rumination, even thought and rumination within the divine and the sacred, is grounded in existence and is proved strong by this action. The effort in every good deed of existence, the tireless effort in fulfilling duties and overcoming yourself in all directions, which requires such an effort, is then truly the greatest sacrifice of

> faith and gratitude you can bring to the Creator of the world. One's gifts and dispositions. In fact, this is the most important saint prayer which can rise from the deep strength of his believing heart and loving God, your Father (PESTALOZZI, 2009, p. 152).

Thus, he affirmed the idea that from the rumination and reading of the world with observation, manipulation and experimentation of things, the child would construct concepts. Thus, the art of Pestalozzian elementary instruction would also be present in the selection of suitable objects for active observation. In these terms, the number, the form and the word are the pillars of support and origin of all knowledge of man (SOËTARD, 2010).

For Rohs (2010, p. 69), Pestalozzi in the Sixth Letter emphasizes that "number, shape and language are at the same time the means of elementary education as the sum of other external properties, of an object that meets in the circle of its contour and the relations of its numerical properties, and that are assimilated by my conscience through the language".

Numbers are, in essence, abstractions of magnitude; therefore, it is necessary to precede or accompany, at the same time, the elements of arithmetic with the initial foundations of drawing, this is experimentation. For him, the intuitions and achievements of the first divisions of the number with representations, enlarged or diminished, of real objects; then, with determined points, in which the child does not accept the real forms as a number, but which can prove and experience in the reality of the points even of the reality of numerical relations.

For Mesquida (2016), Pestalozzi

> [...] feeds and fulfills the integrality of the human being: mind, feeling, action. Or, still, intellect, feeling and action. Modernly, it could be translated by the formula knowing how to think, knowing how to feel, knowing how to act, but knowing how to think is nourished by feeling as well as knowing how to act has its dynamic source in the heart (MESQUIDA, 2016, p. 21).

Pestalozzi, without a doubt, gave a central place to the number and its learning. He proposed new ways of approaching the teaching of Arithmetic and his students obtained surprising results in the learning of calculus (GALLEGO, 2005).

For Pestalozzi, "Arithmetic, in itself, brings in its entire origin the ability to add and remove unity/number of various things from nature" (PESTALOZZI,

1980, p. 176)[9]. For him, the art of knowing an object from the representation of quantity (of number) is nothing less than an abbreviation of calculation.

Pestalozzi considered the number as one of the three elementary means for obtaining knowledge and, for this reason, devoted special attention to its learning. While the word and the form need the number to present themselves as clear intuitions, it considered the number as the only means that has no subordination.

And, according to Costa (2014), the first exercises indicated for children consisted of asking for the number of multiple collections: parts of the body, drawn objects, fingers, stones, objects that have the hand. Thus, it was proposed that children solve these questions by counting, which they learned by imitation, seeing adult people, in particular the mother, counting different proposed collections.

The syllabication boards were also used as collections and, in this way, the learning of numbers was combined with that of words. The learning of words was based on numbers, because the child was asked about the number of syllables of each word and the pronunciation of which they occupied the first place, the second place, among others. Here appears another type of exercise whose answer is an ordinal number, which was also helped by counting in this context.

The first collections had a maximum of ten elements and different exercises were performed on them, which were intended to be an introduction to arithmetic operations. Subsequently, when the student understood the additions of one and one unit up to ten and when he learned to express them easily, the questionnaire was resumed, but the question varied:

> When you have the boards, how many times do you have a board?"- The child takes aim, answers and answers exactly: "When I have the boards, I have two times one board. How many times one is two? How many times one is three? etc. How many times are there one in two? And in three? (PESTALOZZI, 1980, p. 107).

As soon as the student knew the simple and elementary form of addition, multiplication and division and had become familiar, through intuition, with

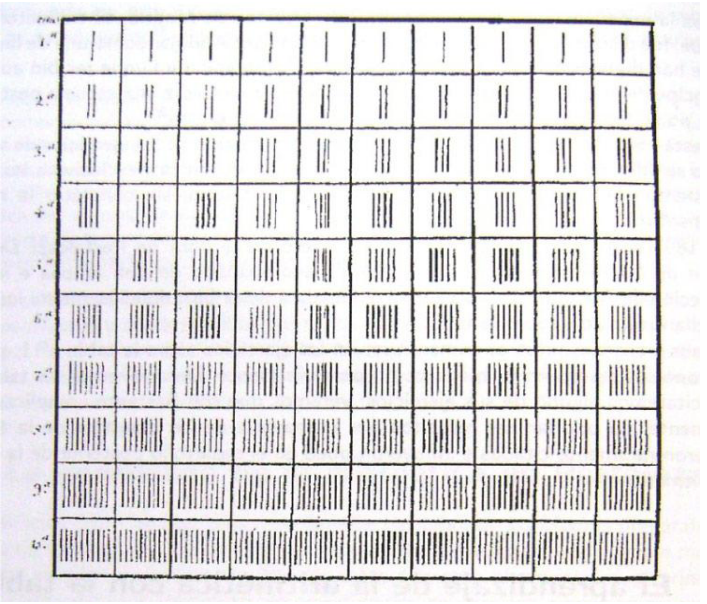
9 In the book First Lesson of Things (1861), from Norman A. Calkins, the methodological composition is anchored in the educational principles of the method of intuitive teaching from Pestalozzi. Oliveira (2015) was dedicated to the analysis of how happened the teaching of numbers from the pedagogical ideas systematized by Pestalozzi (from which were appropriated and incorporated by Calkins in the composition of his pedagogical manual).

the nature of the forms of calculation, he tried to do, through another exercise, to know and make the original form of subtraction:

> If you take one of the boards that have been added and asked: 'When you take one out of ten how many are left?' - The child did not answer and replied: 'When I take one out of ten, there are nine left'. Then the second board is taken and it's asked: 'One taken from nine, how many are left?' - The child counts again, finds eight and answers: 'One taken from nine are eight.' And as such it goes until the end (PESTALOZZI, 1980, p. 107).

The exercises on the first ten numbers were included by Pestalozzi in the *Libro de las Madres* and therefore corresponded to teaching children in their games. We emphasize that, as the *Libro de las Madres* was used in the school of Pestalozzi, "the exercises commented on possibly should be the first ones that the students performed when starting their education" (COSTA, 2014, p. 47).

FIGURE 3 – BOARD N. 1 BY PESTALOZZI. NUMBERS UNTIL A HUNDRED, REFERRING TO THE IDEA OF THE RELATIONS THAT EXIST BETWEEN THEM.



SOURCE: Chavannes (1809, p. 204).

Through the use of this board, eight different types of exercises can be performed, which must be addressed successively, as they form graduation. In all of them, the board is used in rows, considering the different relationships between the quantities that the scribbles represent.

The use of this board admitted that the child exercised the ability to see the line as a unit and to establish the relationships between these units. Oliveira (2017) tells us that, to reach the first objective, it was necessary to ask the child to reproduce four lines on the blackboard and write the number 4; five lines and write the number 5, and so on, because "reproducing lines was also the time to write numbers" (OLIVEIRA, 2017, p. 1026). Besides, observing this table, the teacher could ask questions involving operations of addition, subtraction, multiplication and division, mentally, without the need to register the signs (+, -, ×, ÷). On this, Chavannes (1805) gives us an example:

> [...] 'how many 37 times one is 5?' By looking at the board, the child would construct the answer. Let us know how: she would go to the fifth row and count 7 times the squares with 5 strokes that are 35; to form 37 times 1, she would advance two more lines in the next box (CHAVANNES, 1805, p. 32).

For Pestalozzi, this was a simple and elementary form of calculation, so that the child, according to Chavannes (1805), would perform it in stages. In the beginning, the student is asked to imitate a teacher, who performs the exercise first, and recognizes that the first row of the board consists of collections of a scribble, the second row of collections of two scribbles, the third row of three and, as such, successively.

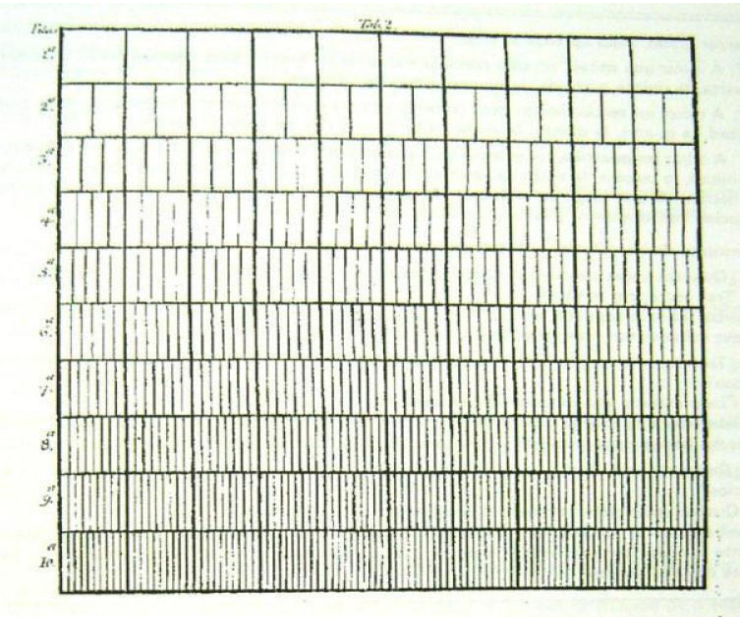
After this exercise has been done in a row, for example, in the third, another is requested, which consists of reading the row as a collection of three elements, that is, one times three, two times three, four times three. These exercises are performed from the imitation of the teacher, aided by counting.

Thus, as Oliveira (2017) points out, the elements of Geometry for Pestalozzi

> [...] they were studied from the form and the measurement, as well as the linear drawing and the artistic drawing being independent subjects, but subordinated and articulated to the first ones. Through the intuitive method the child began to study the elements of Geometry from the form (OLIVEIRA, 2017, p. 1019).

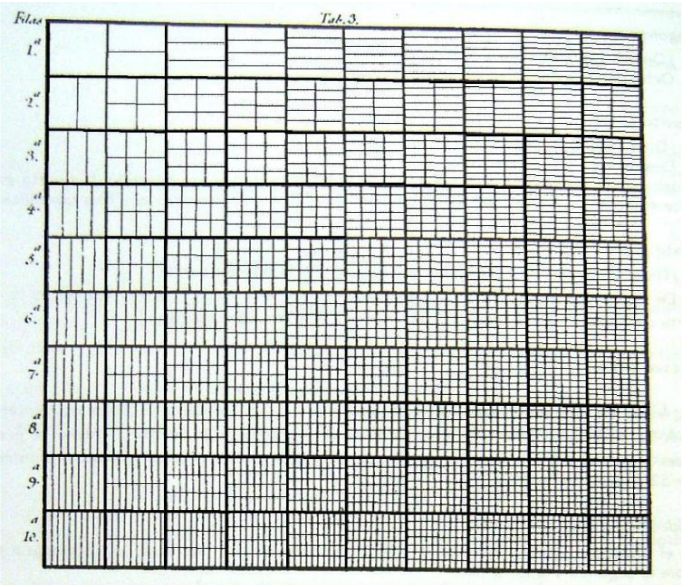
The other exercises presented in the text by Chavannes (1809) have a higher degree of difficulty, but they always take the lines and houses as units.

FIGURE 4 – BOARD N. 2. BY PESTALOZZI. TEN ROWS WITH TEN EQUAL SQUARES EACH. THOSE IN THE 2ND ROW ARE DIVIDED INTO HALVES BY A VERTICAL LINE, WITH THOSE IN THE 3RD ROW IN THIRDS, THOSE IN THE 4TH ROW IN QUARTERS AND SO ON LIKE THIS UNTIL THOSE IN THE 10TH ROW, WHICH ARE DIVIDED INTO TENTHS, ALWAYS THROUGH VERTICAL LINES.



SOURCE: Chavannes (1809, p. 206).

FIGURE 5 – BOARD N. 3 BY PESTALOZZI IS AN EXTENSION OF TABLE NO. 2. IT CONTAINS 10 ROWS OF 10 SQUARES DIVIDED BY VERTICAL LINES AND THE SQUARES ARE DIVIDED INTO EQUAL PARTS BY HORIZONTAL LINES, BEING: THOSE OF THE SECOND COLUMN IN TWO PARTS, THOSE OF THE THIRD COLUMN IN THREE, UNTIL THOSE OF THE TENTH COLUMN THAT ARE DIVIDED INTO 10 EQUAL PARTS.



SOURCE: Chavannes (1809, p. 208).

Costa emphasized that,

> On this board eight different types of exercises were performed. Pestalozzi's proposal, especially with regard to n 2 and 3, it was too ambitious for the place that Arithmetic was given in school, which may explain why these exercises were not generalized. According to Gallego (2005), the understanding of the exercises that Pestalozzi proposed for tablets no. 2 and 3 were beyond the reach of most first-letter teachers, who only came to know some operations with whole numbers (COSTA, 2014, p. 51).

On the other hand, in Oliveira (2017), we found that the child, when already familiarized with the exercises proposed in table 1, advanced to the parts of the units, that is, studied the fractions. Pestalozzi (2009, p. 46) tells us that "The straight line is still the simplest and the easiest way here, the one that lends itself best to observation and execution", as, according to Oliveira (2017, p. 1027), it can be seen in the figure that "Dividing a straight line into equal measures is to make the child observe the parts or fractions of a unit". Soon the fractions also had a table, just like in the study of numbers. The analysis of table 2, both vertically and horizontally, shows us that

> [...] the notion of whole and fractional unit can also be elucidated by the idea of space. For example, looking at it from the vertical direction, it is noticed that from top to bottom the spaces between the parts are decreasing. From the bottom up, they are increasing. In addition to other learnings, this table II allows the child to learn, through each box, how many equal parts are necessary to compose a unit. As the study progresses, fractions are represented by numbers (½;⅓,¼,...) (OLIVEIRA, 2017, p. 1027).

In this way, the Swiss pedagogue was inspired by the pedagogical proposal of Comenius (Orbis *sensualium* pictus, 1658), defender of the universalization of education. Like him, he will indicate the use of the image, as art and as a methodological strategy, to teach the child reading and writing, as an important didactic "technique" (MESQUIDA, 2016).

Thus, we can see that in the teaching of Geometry (straight line, angle, point), the method indicated by Pestalozzi, already made associations with arithmetic, because as we have seen, through figures and lines (drawing), the concrete is exalted, we sought to explain the mathematical calculation, going beyond the relationships between shapes and measures that Geometry brings us, but associating it with other elements that surround this content, such as the calculation with numbers, multiples and submultiples of the number, the system of decimal numbering.

A look at teaching and mathematics in Montessori

Montessori was born in Italy in 1870 and died in the Netherlands in 1952. From an early age, he was interested in science and studied medicine at

the University of Rome. Later, when he specialized in psychiatry, he became interested in studies about children with mental retardation. According to Foschi (2012), Maria Montessori was a doctor, psychiatrist, anthropometric, "specialist" in experimental psychology, teacher, politician, feminist, theosophist, laywoman, Catholic. And, each of those fields, in a certain sense, have elements that we find in her applications and her method.

Influenced by Pestalozzi, Maria Montessori devoted herself to education and systematized a method, Montessori Method (1907), in which affection and affectivity are the basis for the Montessori Method.

For Foschi (2012, p. 13), one of the keys to reading the Montessori Method lies in the "fundamental role that behavioral sciences, including experimental sciences, have played in the social management of modern liberal societies". In this sense,

> [...] since the first courses on her Method, given since 1909, [Maria Montessori] dedicated the initial cycle of lessons to psychology and anthropology, emphasizing that these sciences, which are not linked to the abstract laboratory, but must lead to a new pedagogy consistent with scientific research. In fact, from reading the Method, one can deduce an original use of the laws related to the physiology of the senses and the use of experimental science theories, especially with the aim of methodologically training teachers (FOSCHI, 2012, p. 13).

The Montessori method is biological because it is based on scientific information about child development. In this method, the children conduct their learning and the teacher is attentive to detect the particular way of each one (MONTESSORI, 1965; 2013). *Pari passu*, the method[10] goes from the concrete to the abstract, so that the educator developed suitable teaching materials to provoke the child's reasoning, through the mediation of the teacher.

For Costa (2001), the method is "active, as it gives importance to work: children must take care of their hygiene and cleaning of the rooms. It strives to

10 For Montessory (1965, p. 42), the method of observation will be founded upon a single basis: "the freedom of expression that allows the children to reveal to us their qualities and necessities, that would have remained hidden or repressed in an infense environment to spontaneous activity. Thus, it is necessary that, simultaneously to the observer, also coexists the object to be observed; and if, in a way, it is shown a preparation so that the observer can glimpse and recollect the truth, in another, urges to predispose the conditions that make possible the manifestation of natural characters of the child".

individualize teaching, encouraging concentrated free activity and the principle of self-education" (COSTA, 2001, p. 2).

Regarding Maria Montessori's relationship with Mathematics and teaching, we highlight the publication of the works: Psychogeometry (1934) and Psychoaritmethics (1934) which display principles for the teaching of Mathematics and pedagogical materials (COSTA, 2001).

For the Montessori method (2013), the use of concrete materials and the teaching of mathematics would be inseparable and from this relationship would come the principles of abstract-concrete and intuition in the process of building demonstrations.

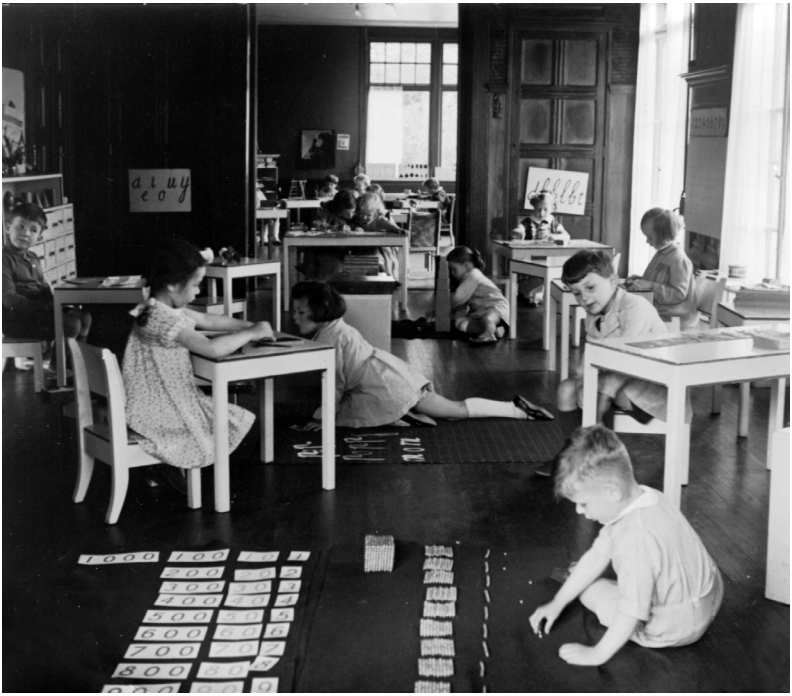
The use of concrete materials would awaken the student's inventive potential and ally him with the desire to know and lift the veil of the universe that is still unknown to him. For Röhrs:

> The teaching material also had the function of helping the child to 'grow in peace' so that they acquired a high sense of responsibility. This material, which constituted one of the elements of the 'prepared environment' of the children's home, was methodically designed and standardized, so that the child who had freely chosen to take care of one of the proposed objects was located in a previously determined situation and was guided, without knowing it, to face its intellectual design (RÖHRS, 2010, p. 21).

Among the teaching materials developed by Maria Montessori, one of the most used for teaching mathematics is the golden material. Falzetta (1997, p. 1) describes the golden material[11] as constituted by 500 cubes of 1 cm x 1 cm x 1 cm, which represents the unit; 100 bars of 1 cm x 10 cm x 1 cm, representing the ten; 10 plates of 1 cm x 10 cm x 10 cm and 1 cube of 10 cm x 10 cm x 10 cm, which represents the unit of thousands.

11 Today, some educators prefer to use another denomination that does not bind itself to the represented value, such as the terms "little cube" (unit), "bar" (ten) and "big cube" (thousand) (FALZETTA, 1997, p. 1).

FIGURE 6 – MONTESSORI METHOD. CHILDREN USING GOLDEN MATERIAL.



SOURCE: Available on: https://sinteno.weebly.com/blog/nova-educacao-na-pratica-metodo-montessori. Access on: jan. 4, 2019.

For Oliveira and Bortoloti,

> Through these materials it is possible to attract the child's intelligence to things and phenomena, in addition, the presentation of the material can be individual without the need for a verbal lesson. It should be presented slowly, clearly so that the child can by themself, perceive the movement and execute it correctly for as long as they want. The eventual errors, caused by the lack of experience, most of them, must be corrected by the child, except in situations of misuse, disorderly. In that case, the teacher can drive them to make the appropriate use (OLIVEIRA; BOTOLOTI, 2012, p. 416).

This material helps in the teaching and learning of Mathematics, with children who had learning difficulties. Given the success of the golden material, Brazilian teachers started using it in the classroom to work on the structures of the Decimal Numbering System, the algorithms of the four fundamental operations, geometric concepts, fractions, decimal numbers, percentage, areas and volumes.

In traditional education, children end up "mastering" the algorithms from tiring training, but without being able to understand what they do. With the golden material, the situation is different: the abstract numerical relations have a concrete image, facilitating the understanding. Then, in addition to understanding the algorithms, there is a remarkable development of reasoning and a much more pleasant learning experience.

It was known as "Material of the Golden Beads" and its shape allowed the children themselves to produce the tens and hundreds. The inaccuracy of the measurements of squares and cubes was a problem when carrying out activities with decimal numbers. For this reason, Lubienska de Lenval modified, building in wood, in the form we find today. It is important because the abstract numerical relations start to have a concrete image, facilitating the understanding, the development of logical reasoning and a much more pleasant learning experience.

It allows students in the early years of elementary school to better understand the operations of addition with exchanges and subtraction with grouping, in addition to enabling them to still have some difficulty in understanding the passage from abstract to concrete, learning with understanding and more effectiveness.

As an example, the teacher can mediate activities by asking: What is a decimal number? What fraction of the integer does each little cub represent? How many hundredths make up an integer? How many hundredths do we need to make 1 tenth? How many hundredths do we need to make 8 tenths? With 1 tenth, how many hundredths can we make? How many tenths have 2 integers? In two tenths, how many hundredths are there?

Also, it is possible to organize activities to compare decimal numbers, so that he understands the positional value of the figures, representing the values indicated with the golden material. It is also plausible to perform operations with decimal numbers, using the Material, so that the student can represent the values indicated in each operationby their decimal fractions, and then they must be represented with the golden material and, finally, calculated. Example: $1.3 + 3.02 = 4.32$

Finally, Maria Montessori, with the object of study about the child's learning, scientifically researched, because "she looked for the best possible way to help the child to develop positively in all dimensions [...] observing the child in different situations anddifferent stages of their life" (DUBUC, 2010, p. 204).

Maria Montessori considered fundamental the communication between human beings so that the attention given to the child is done with love, since "without love, life is not possible" (DUBUC, 2010, p. 208).

She also considered that as adults we should recognize four periods of human development (from 0 to 24 years old). In the first period (0 to 6 years) exploration is sensory, then imagination comes to the fore. The educator was criticized, but her theory attracted many, with the simplicity of using didactic material with children, so that in Montessori Pedagogy, "children are active and teaching is open" (DUBUC, 2010, p. 216).

Final considerations

The views we present in this article about pedagogy with numbers in the thinking of Comenius, Pestalozzi and Montessori showed us that mathematics, in the educational thinking of these intellectuals, was approached as a language system that, instead of letters and words, uses numerical symbols. They also showed that there is a circulation of ideas among the educational thinking of the three teachers, about the teaching of concrete for the abstract and the use of didactic materials in the teaching of Mathematics.

It is worth mentioning that in the methods proposed by Comenius, Pestalozzi and Maria Montessori, associates Mathematics with practical and gradual activities, so that the active participation of children is valued, their subjective growth, within the process of building their learning.

Therefore, these methods need to be disseminated, studied in the formation of the mathematics teacher, even so, that he answers how to teach mathematics, who teaches mathematics and which mathematics is taught, in the perspective that we understand that the Pedagogy of Comenius, Pestalozzi and Montessori they are not over, not least because each one of us, whether future Mathematics teachers or Pedagogues, has as one of the teaching objectives, what Montessori said: "rediscover the secret of childhood and build from the child".

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