PRELIMINARY GEOID MODEL IN SAN JUAN PROVINCE: A CASE STUDY IN THE ANDES

Modelo preliminar do geóide na província de SanJuan: Estudo de caso nos Andes

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ABSTRACT

A high-resolution and high-precision detailed gravimetric geoid has been computed for San Juan province in Argentina, ranging from 27º S to 34º S in latitude and 72º'W to 65º W in longitude. The gravimetric geoid was calculated using the RTM method, a multiband spherical Stokes Fast Fourier Transformation, and the removerestore technique for the spherical harmonic reference field and the terrain.

As an external evaluation, the gravimetric quasigeoid/geoid was compared to the geoid heights obtained from 90 GPS/levelling points available for the province. Finally, a GPS-tailored local geoid, which fits the GPS observations, was computed. **Keywords:** Gravimetric Geoid; RTM Method; High Precision Geoid.

RESUMO

 Um geóide gravimétrico, com alta resolução e alta precisão, foi calculado para a província de San Juan na Argentina, delimitado pelos paralelos 27º S to 34º S e pelos meridianos 72º'W to 65º W. O geóide gravimétrico foi calculado usando o método RTM associado com uma transformação rápida de Fourier de múltiplas bandas sobre a função esférica de Stokes, com aplicação da técnica *remove-restore* para o modelo de referência do campo da gravidade e de elevação do terreno, ambos expressos em harmônicos esféricos.

Como avaliação externa, o quase-geóide/geóide foi comparado com o geóide obtido de 90 pontos de nivelamento disponíveis na província e que foram ocupados também com GPS. Finalmente, foi calculado um geóide local adaptado via GPS, o qual se ajusta com as observações GPS.

Palavras-chave: Geóide gravimétrico; Método RTM; Geóide de alta precisão.

1 INTRODUCTION

 The main purpose of this paper is to compute a gravimetric geoid model for San Juan province, one of the roughest areas of Argentina.

 The geoid is an equipotential surface of the Earth's gravity field that most closely *approximates* the mean sea surface at rest*.* At every point, the geoid surface is perpendicular to the local plumb line. It is then a natural datum or reference surface for orthometric heights measured along the curved plumb lines and, at the same time, the geoid is the best graphical representation of one equipotential surface of the Earth gravity field. The geoid heights are used to convert GPS-derived ellipsoidal heights (h) to orthometric heights (H).

 An area covering San Juan province was chosen in this study due to the rough topography, the presence of GPS/levelling data and sparse gravity coverage coming from different sources. One view of the referred area is given in Figure 1.

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 All computations outlined in the present paper have been done by programs of the GRAVSOFT package: a set of routines for gravity field modelling developed by the Geophysical Department of the University of Copenhagen and Kort og Matrikelstyrelsen (KMS), nowadays the Danish National Space Center (DNSC) (TSCHERNING et al., 1992).

 The theoretical background related to the estimation of the gravimetric geoid model will be outlined in next sections together with the description of the data available in the area under study. Finally, some numerical studies carried out in this research will be presented.

2 USED DATA

 This section will describe the San Juan province data and what pre-processing has been done before computing the regional geoid model.

2.1 Surface gravity measurements

 The point gravity measurements, provided by different sources, were referenced to the International Standardisation Net 1971 (IGSN71).

 A total of 4922 measured gravity points were used in this work. The mean data spacing of is approximately 6 km, but this is lower in areas of commercial interest and higher in areas where it is difficult or impossible to access to collect ground gravity data like on the Andes mountains. The distribution of the gravity data is presented in Figure 2, where the grey scale indicates the topography.

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2.2 Gravity anomalies

 Free-air gravity anomalies were calculated using the parameters of the Geodetic Reference System 1980 (GRS80). The point free-air gravity anomalies were calculated using the following formula:

$$
\Delta g_{FA} = g + \delta_{g_{atm}} + \frac{2\gamma_a}{a} \left[1 + f + m + (-3f + \frac{5}{2}m)\sin^2\varphi \right]H - 3\frac{\gamma_a}{a^2}H^2 - \gamma \tag{1}
$$

where:

$$
\Delta g_{FA} = g + \delta g_{atm} - \gamma + 0.30877(1 - 0.00142 \sin^2 \varphi)H - 0.75 10^{-7} H^2 \text{ [mGal]} \tag{2}
$$

 This formula uses the second order free-air reduction, applies atmospheric correction (δg_{atm}) and evaluates normal gravity γ with Somigliana's closed formula, using the parameters of the GRS80. The atmospheric correction is computed as follows (TORGE, 1989):

$$
\delta g_{atm} = 0.874 - 0.99 \, 10^{-4} \, H + 0.356 \, 10^{-8} \, H^2 \, [\text{mGal}] \tag{3}
$$

where the height H is in m.

The simple planar Bouguer gravity anomalies were computed as:

$$
\Delta g_{Bouguer} = \Delta g_{FA} - 2\pi K \rho H \tag{4}
$$

where ρ is the density of the topography mass, K is the gravitational constant and H is height of the gravity points. The simple Bouguer anomaly was used to estimate the quasigeoid minus geoid separation on land.

With a standard density $\rho = 2.67 g / cm^3$, the Bouguer gravity anomaly is:

 $\Delta g_{B_{\text{parameter}}} = \Delta g_{F_A} - 0.1119H \text{ [mGal]}$ (5)

 A carefully visual inspection for identification and removal of data blunders and duplicate points was carried out after the gravity anomalies computation. With basis in this approach, a total of 3103 points were eliminated. The statistics of the free-air and Bouguer gravity anomalies is shown in Table 1.

	mın	max	mean	std dev
Free-air gravity anomalies	-145.50	229.84	-58.73	70.44
Simple Bouguer gravity anomalies	-415.25	43.31	-171.60	71.50

Table 1- Statistics of gravity anomalies in the area under study. Unit:[mGal].

2.3 KMS02 free-air gravity anomalies

 KMS02 2′×2′ altimetry derived free-air gravity anomaly field (ANDERSEN et al., 2005) has been used to fill in information in the Pacific Ocean neighbouring area, with the purpose to improve the quality and accuracy of the geoid. The statistics of the gravity anomalies for KMS02 grid can be seen in Table 2, after land gravity anomalies were removed using the *grdlandmask* option in GMT (WESSEL and SMITH, 1998).

Table 2 - Statistics of the gravity anomalies derived from satellite altimetry.

Unit:[mGal]

2.4 Digital elevation model

 Digital Elevation Models (DEMs) are essential for obtaining a good gravimetric geoid model in mountainous areas like the one presented here.

The global digital elevation model SRTM30 DEM (JPL, 2006), with a horizontal grid spacing of 30 arc- seconds (approximately 1 kilometre) covers the area 33.9958ºS-27.0042ºS; 71.9958ºW-65.0042 ºW. The statistics of the SRTM30 DEM data and the statistics of the height of the gravity stations in the study area are given in Table 3.

	mın	max	mean	std dev
SRTM30 DEM		6813		1388
H (gravity stations)		490.	897	706

Table 3 - Statistics of the SRTM30 DEM in the area under study. Unit:[m].

2.5 Global Gravity Model

 A global gravity model (GGM) describes the long wavelength characteristics of the earth's gravity field. The reference gravity field was computed in this paper from a composite EGM/GRACE model, where GGM02S (TAPLEY et al., 2005) is used from degree *n*=2 to 99 and order *m*=0 to 99, and EGM96 (LEMOINE et al., 1998) from degree and order *n, m*=100 to 360. We will refer to this model as GGM02SEGM96*.* From the contribution of the composite model GGM02SEGM96 a reference gravity anomaly (Δg_{GM}) grid and a reference geoidal undulation (*NGM*) grid were calculated using program "harmexp*"* from GRAVSOFT.

The gravity anomaly estimated at a position (ϕ_P , λ_P) is expressed in spherical approximation as:

$$
\Delta g_{GM} = G \sum_{n=2}^{n=n_{max}} (n-1) \sum_{m=0}^{n} \left(\overline{C_{n,m}} \cos m \lambda_P + \overline{S_{n,m}} \sin m \lambda_P \right) \overline{P_{n,m}} (\sin \phi_P)
$$
(6)

and the reference geoidal undulation as:

$$
N_{GM} = R \sum_{n=2}^{n=n_{max}} \sum_{m=0}^{n} \left(\overline{C_{n,m}} \cos m\lambda_P + \overline{S_{n,m}} \sin m\lambda_P \right) \overline{P_{n,m}} (\sin \phi_P)
$$
 (7)

where *G* is the mean gravity of the Earth, *R* is the mean radius of the Earth, $\overline{C_{nm}}$ and $\overline{S_{n,m}}$ are the fully normalized spherical harmonic coefficients of the disturbing potential, $\overline{P_{nm}}$ are the fully normalized associated Legendre functions (HEISKANEN and MORITZ, 1967), and n_{max} denotes the maximum degree and order of expansion of the geopotential solution.

 Both grids were computed in the test area. They are shown in Figures 4 and 5; the corresponding statistics can be seen in Table 4.

Figure 5 - GGM02SEGM96*.* Geoid heights grid.

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 $\frac{18}{18}$ $\frac{16}{14}$

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	min	max	mean	std dev		
Geoid [m]	11.731	45.359	27.779	6.307		
Gravity anomalies [mGal]	-202.27	233.85	32.55	71.86		

Table 4 - Statistics of GGM02SEGM96 geoid heights and gravity anomalies in grid form.

Figure 4 - GGM02SEGM96*.* Gravity anomalies grid.

2.6 GPS and levelling data

 The accuracy of the gravimetric geoid undulations can be evaluated by two methods: one is the external comparison with geometrical geoid undulations from GPS and spirit levelling and the other is the internal propagation of data errors. For the first method, points with GPS-derived ellipsoidal heights and orthometric heights with respect to a local datum constitute an important type of data. They can be incorporated in the database in order to determine discrete precise geoid undulations by the geometrical approach. Geometrical geoid undulation on land can be determined, for the absolute cases, by:

$$
N^{GPS} = h^{GPS} - H^{levelling} \tag{8}
$$

 It should be pointed out that these geoid heights, opposed to the geoid heights determined from global models and gravity data, refer to a local vertical datum. GPS/benchmark height information on 101 points across San Juan province has been collected. This data includes geodetic latitude and longitude, ellipsoidal heights and levelling heights. The ellipsoidal heights are referred to the World Geodetic System 1984 (WGS84). Two GPS/levelling networks were used for the

external evaluation of the gravimetric geoid accuracy. One network is the PASMA (21 points) and the other RURAL (81 points). The distribution of GPS/levelling points in San Juan province is shown in Figure 3. Rural GPS-levelling network (HERRADA et al., 2004) was carried out by collecting GPS measurements at selected benchmarks, which are ∼3 km apart, using double frequency receivers. PASMA network was established by government agencies for the purpose of improving the infrastructure for the mining activity in Argentina. The GPS observations were performed with six dual frequency receivers with baseline length ranging from about 60 to 120 km (GILLONE and BRUNINI, 1999).

 Before all these GPS/levelling points can be used for comparisons, it was necessary to clean the data (identify outliers and blunders). A 2D contour map of the geometric geoid was plotted and after a visual inspection test, a total of 11 points were identified as blunders and were eliminated from the original database of the RURAL network and 1 point was eliminated from the PASMA network. After the suspicious observations were removed, the final GPS/levelling data in Argentina, consist of 70 GPS/levelling points (RURAL network) and 20 GPS/levelling points (PASMA network).

	min	max	mean	std dev
PASMA (20 points)	23.41	34.84	28.56	3.69
RURAL (70 points)	23 77	25.93	24.94	0.51

Table 5 - Statistics of the GPS/levelling-derived geoid. Unit: [m].

3. COMPUTATIONAL METHODOLOGY

The San Juan precise geoid has been computed in two steps:

1- A gravimetric quasigeoid/geoid model, computed by spherical FFT in a global datum, and

2- A GPS-tailored local geoid, which fits the GPS observations.

 The computation of the gravimetric geoid model was based on the classical remove-predict-restore technique. The underlying procedure can be summarizes as follows:

a) *Remove* gravity anomalies computed from the GGM02SEGM96 global gravity model and RTM terrain effects from the observed free-air gravity anomalies.

 The *GGM02SEGM96* global gravity model values were computed in a grid (GRAVSOFT program "harmexp" program). From this grid, gravity anomalies values were linearly interpolated and removed from the observed free-air anomaly gravity points using ("geoip" program). Terrain effects have been also removed in a consistent residual terrain model (RTM) data reduction using prisms ("tc" program), taking into account the topographic irregularities relative to a mean height surface for details see Forsberg (1984). A 100km resolution of the RTM mean height surface was chosen, computed by ("tcgrid" program).The statistics of gravity data

reductions are shown in Table 6. In this step residual gravity anomalies (^Δ*gresidual*) were obtained.

	min	max	mean	std dev
Δg_{FA} (land + KMS02)	-224.40	229.84	-24.19	73.74
Δg _{GM}	-200.08	198.22	-30.14	68.49
$\Delta g_{FA} - \Delta g_{GM}$	-313.52	206.72	-24.05	48.95
Δg_{RTM}	-500.08	194.10	-16.20	38.95
$\Delta g_{residual} = \Delta g_{FA} - \Delta g_{GM} - \Delta g_{RTM}$	-105.75	380.33	-7.85	36.13

Table 6 - Statistics of 8354 gravity points. Unit: [mGal].

Compute residual undulations ($^{N_{\Delta g}}$ *residual*) by a spherical representation of Stokes's convolution integral using the residual gravity anomalies ($\Delta g_{residual}$). The use of FFT requires that the random residual gravity anomalies must be interpolated on a grid and then they are converted into residual height anomalies by spherical FFT. The residual gravity anomalies were gridded by least-squares collocation ("geogrid" program), and a 100% zero padding was used to limit the periodicity errors of FFT ("spfour" program). A Wong-Gore kernel modification has been used for spherical harmonic degrees less than 80 to limit the long-wavelength errors. The collocation gridding was done using a correlation length of 25 km assuming a freeair anomaly noise of 2 mGal.

In principle, the RTM method evaluates the quasigeoid and the Molodensky integral is used to obtained residual quasigeoid heights.

$$
\zeta_{\Delta g} = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g + g_I) S(\psi) d\sigma \tag{9}
$$

where g_1 is the first term of the Molodensky series for the terrain-reduced field, and ζ_{Λ} is the residual quasigeoid. The integral (9) is, in practice, identical to Stokes's integral, as the g_1 -term is very small, and may for most practical purposes to be neglected.

 In the used FFT method, the Molodensky/Stokes' integral is written as a spherical convolution in latitude and longitude for a given reference parallel (ref), and by utilization of a number of bands a virtually exact convolution expression may be obtained by a suitable linear combination of the bands. For each band the convolution expression is evaluated by:

$$
\zeta_{\Delta g} = \Delta g(\varphi, \lambda) \sin \varphi * S_{ref}(\Delta \varphi, \Delta \lambda) = F^{-1}[F(\text{Sref })F(\Delta g \sin \varphi)] \tag{10}
$$

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where S_{ref} is a modified Stokes' kernel function, and * and F the two-dimensional convolution and Fourier transform, respectively - for details see Forsberg and Sideris (1993).

 To evaluate the multiband spherical FFT, the "spfour" program was used. The gravimetric residual quasigeoid has been computed in the region 27º to 34º in latitude and 65ºW to 72ºW in longitude at a 1.5' resolution in latitude and longitude. b) *Restore* terrain effects and GGM02SEGM96 for the final quasigeoid

 The topographic RTM restore signal was evaluated by FFT methods, using the first-order mass-layer approximation to the RTM geoid effect ("spfour" program). Table 7 shows the statistics for the restore steps.

	min	max	mean	std dev
$\zeta_{\Delta g}$	-2.49	7.21	0.10	0.95
$\zeta_{GGM\,02SEGM\,96}$	11.73	45.36	27.88	6.19
ζ_{RTM}	-0.39	0.81	0.00	0.09
$\zeta = \zeta_{\Delta g} + \zeta_{GGM02SEGM96} + \zeta_{RTM}$	11.66	45.79	27.98	6.16
$N-\zeta$	-2.23	0.05	-0.20	0.35
N	11.66	45.74	22.77	5.97

Table 7 - Statistics of the restore step. Unit: [m].

c) *Convert* the quasigeoid to geoid

 The RTM method yields the quasigeoid. In order to obtain the geoid, the reparation quasigeoid minus geoid is needed. The quasigeoid is related to the geoid by the following formula (HEISKANEN and MORITZ, 1967):

$$
\zeta - N = \frac{\Delta g_{Bouguer}}{\bar{y}} H
$$
\n(11)

where $\Delta g_{\text{Rouvner}}$ is the Bouguer anomaly and $\bar{\gamma}$ is the mean normal gravity.

 The statistics of the final geoid grid can also be seen in Table 7. The difference between quasigeoid and geoid is approximately 35 cm in terms of standard deviation in San Juan. The difference between ζ and N is significant and is strongly correlated with the terrain; but as the levelling data have not been adjusted

in geopotential numbers, the theoretical type of geoid is not an issue, especially considering that GPS-levelling is used to constrain the geoid in the end.

4. COMPARISONS AT GPS BENCHMARKS

 The accuracy of the computed model was assessed through comparisons with interpolated values of the gravimetric quasigeoid/geoid at both networks of GPS/levelling points and for all GPS/levelling points. A total of 80 GPS/levelling points were used as external control for the quality of the gravimetric quasigeoid/geoid solution.

 Table 8 shows the statistics of the absolute differences between the GPS/levelling-derived geoid and the estimated quasigeoid/geoid solutions. The first three rows show the statistics of the differences between the *GGM02EGM96* geoid at the same GPS/levelling points.

$[m]$.					
	min	max	mean	std dev	
$N^{GGMO2EGM_n.gri}$ - N^{GPS} PASMA	-0.49	4.03	1.35	1.28	
$N^{GGMO2EGM_n.gri}$ - N^{GPS} RURAL	0.61	1.37	0.97	0.20	
$N^{GGMO2EGM_n.gri} - N^{GPS}$ ALL	-0.49	4.03	1.06	0.66	
\mathcal{L} - N ^{GPS} PASMA	0.32	4.54	2.02	1.14	
ζ - N ^{GPS} RURAL	0.29	2.91	1.65	0.80	
\mathcal{L} - N ^{GPS} ALL	0.31	4.54	1.58	0.59	
$N - N^{GPS}$ PASMA	0.34	4.97	2.28	1.24	
N - N ^{GPS} RURAL	0.51	1.96	1.54	0.22	
$N \cdot N^{GPS}$ ALL	0.34	4.97	1.70	0.67	

Table 8 - Comparisons of gravimetric geoid and GPS/levelling-derived geoid. Unit:

5. GPS-LEVELLING FITTING OF THE GEOID

 The computed gravimetric quasigeoid/geoid refers to a global reference system and needs to be fitted to local GPS-levelling data for operational GPS height use, to eliminate datum shift, residual long-wavelength gravity errors, and possible systematic errors in the levelling. The difference between GPS geoid (N^{GPS}) and gravimetric (*Ngrav*) is:

$$
\varepsilon = N^{GPS} - N^{grav} \tag{12}
$$

 Since most countries are interested in using GPS to determine heights in a local vertical datum, to be consistent with existing levelling, the gravimetric geoid has to be tailored to the local level.

The difference ϵ given in (12) was gridded softly using collocation and the final geoid was obtained by:

$$
N^{draped} = N^{gravimetric} + \varepsilon^{grid} \tag{13}
$$

Thus, the draped geoid was consistent with GPS data

The differences between the gravimetric geoid and the GPS geoid was modelled by a smooth function consisted of a trend function f and a *residual* ε*' to* be model by least/squares collocation

$$
\varepsilon = f(\varphi, \lambda) + \varepsilon' \tag{14}
$$

To compute the final geoid, the residual ε' was modelled by least–squares collocation using the second order Markov covariance function using program "geogrid".

$$
C(s) = Co\left(1 + ks\right)e^{-ks} \tag{15}
$$

where *k* is a constant, determined by correlation length, and *s* the distance.

 A correlation length of 100 km and GPS geoid noise (sigma) of 0.05 m was used. Table 9 shows the fit of the GPS levelling data to the tailored geoid "SJGEO".

Table 9 - Fit of GPS/levelling-derived geoid to tailored geoid "SJGEO". Unit: [m].

	min	max	mean	std dev
SJGEO - N^{GPS} PASMA (20)	-0.08	0.09	0.00	0.04
$SIGEO - NGPS RURAL (70)$	-0.11	0.17	0.00	0.06
SJGEO - N^{GPS} ALL (90)	-0.20	0.27	0.00	0.07

The final geoid "SJGEO" was the result of fitting the gravimetric quasigeoid to GPS and it is displayed in Figure 6.

6. CONCLUSIONS

 A centimetre gravimetric quasigeoid/geoid for San Juan province has been computed. The gravimetric quasigeoid/geoid has been computed using gravity data and 1 km x 1km Digital Elevation Model as well as GPS and levelling data combined with GGM02SEGM96 spherical potential coefficients.

GGM02SEGM96 was used to recover the long wavelength contribution of the gravimetric geoid

 The comparison of GPS/levelling geoid heights with the corresponding gravimetric values showed a poor agreement, even though, the effect of the atmosphere, the topography and the ellipticity of the reference surface on the gravity as well as the indirect effect on the computed quasigeoid/geoid has been taken into account.

 The gravimetric quasigeoid solution was fitted to local GPS/levelling data and the "SJGEO" geoid fitted to the GPS/levelling data better than 7 cm.

The preliminary geopotential model (PGM2007A) complete to degree 2160, which it is, at present, being evaluated, could be a good model to improve the long

wavelength in the area under study and help to meet the 1cm geoid, which has been the goal of geodesists and geophysicists.

We can conclude that with the improvement of gravity data coverage, quality and density mainly in the Andes, it will be possible to improve the accuracy of the geoid to meet the requirements needed nowadays for modern geodetic, oceanographic and geophysics applications. The densification of gravity data in the Andes can be carried out with modern measurement techniques like airborne gravimetry. As digital elevations models play an important role in the removecompute-restore technique, the SRTM3 (JPL, 2006) model with a resolution of 3" x 3" has to be evaluated in San Juan province.

REFERENCES

- ANDERSEN O.B., KNUDSEN P., TRIMMER R. (2005): Improved highresolution altimetric gravity field mapping (KMS02 Global marine gravity field). *International Association of Geodesy Symposia, vol. 128, Sanso F (Ed.), Springer. Proceedings of the Symposium 128*: A window on the future of Geodesy, Sapporo, Japan, June 30-July 11, 2003, pp. 326-331.
- FORSBERG R. (1984): *A study of terrain corrections density anomalies and geophysical inversion methods in gravity field modeling*. Report of the Department of Geodetic Science and Surveying no. 355, The Ohio State University, Columbus, Ohio.
- FORSBERG R. AND SIDERIS M.G. (1993): *Geoid computation by multi-band FFT approach*. Manuscripta Geodaetica vol. 18: pp.82-90, no. 2.
- GILLONE R. AND BRUNINI C. (1999): Setting boundaries: A Geodetic Network for Argentinean Mines. *GPS World, vol.10, no.2*, pp.32-37.
- HEISKANEN W.A. AND MORITZ H. (1967): *Physical Geodesy*. W. H. Freeman and Company, San Francisco.
- HERRADA A., MÁRQUEZ R. AND SISTERNA J. (2004): *Modelo Preliminar de Geoide en un Área de 150.000 Has. de la Provincia de San Juan-Argentina y Determinación de Alturas sobre el Nivel Medio del Mar con GPS*. En: Tópicos de Geociencias. Editorial Fundación Universidad Nacional de San Juan. Argentina: pp. 231-254.
- JPL. The Shuttle Radar Topography Mission (SRTM). *The Mission to Map the World .*www2.jpl.nasa.gov/srtm/ Jet l. Accessed in 5 May, 2006.
- LEMOINE F.G., KENYON S.C., FACTIM J.K., TRIMMER R.G., PAVLIS N.K., CHINN D.S., COX C.M., KLOSKO S.M., LUTHCKE S.B., TORRENCE M.H., WANG Y.M., WILLIAMSON R.G., PAVLIS E.C., RAPP H R. AND OLSON T.R. (1998): The development of the joint NASA, GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96, NASA, Technical Publication-1998-206861, July, 1998.

TAPLEY, B., RIES J., BETTADPUR S., CHAMBERS D., CHENG M., CONDI F., GUNTER B., KANG Z., NAGEL P., PASTOR R., PEKKER T., POOLE S., WANG F. (2005): "*[GGM02 - An improved Earth gravity field model from](http://www.csr.utexas.edu/grace/gravity/ggm02/GGM02_JGeod_190_0480.pdf) [GRACE](http://www.csr.utexas.edu/grace/gravity/ggm02/GGM02_JGeod_190_0480.pdf)*", *Journal of Geodesy, vol. 79, no. 8*, pp. 467-478

TORGE W. (1989): *Gravimetry*. Walter de Gruyter Verlag Berlin.

- TSCHERNING C.C, FORSBERG R., KNUDSEN P. (1992): *The GRAVSOFT* package for geoid determination. In Holota P, Vermeer M (Eds), 1st Continental Workshop for the Geoid in Europe: *Towards a precise European geoid in the nineties*, Prague, June 7-9, pp. 327-334.
- WESSEL P. AND SMITH W.H.F. (1998): *New improved version of The Generic Mapping Tools released*. EOS Trans 79(47): 579.

(Recebido em Janeiro/08. Aceito em agosto/08).